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Asymptotic ruin probabilities in a generalized bidimensional risk model perturbed by diffusion with constant force of interest

Qingwu Gao *, Xiuzhu Yang

School of Science, Nanjing Audit University, Nanjing 211815, China

A R T I C L E I N F O

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A B S T R A C T

In the paper, we study three types of finite-time ruin probabilities in a diffusion-perturbed bidimensional risk model with constant force of interest, pairwise strongly quasi-asymptotically independent claims and two general claim arrival processes, and obtain uniformly asymptotic formulas for times in a finite interval when the claims are both long-tailed and dominatedly-varying-tailed. In particular, with a certain dependence structure among the inter-arrival times, these formulas hold uniformly for all times when the claims are pairwise quasi-asymptotically independent and consistently-varying-tailed.

1. Introduction

In this paper, we consider a generalized bidimensional risk model perturbed by diffusion of an insurance company, which launches two classes of insurance business. The model satisfies the following assumptions. For $i = 1, 2$,

A1. the i-th class of claim sizes $\{X_k^{(i)}, k \ge 1\}$ is a sequence of nonnegative, identically distributed, but not necessarily independent, random variables (r.v.s) with common distribution F_i;

A2. the claim arrival process $\{N_i(t), t \ge 0\}$, independent of $\{X_k^{(i)}, k \ge 1\}$, is a general counting process satisfying $EN_i(0) = 0$ and $EN_i(t) < \infty$ for all $0 < t < \infty$;

A3. the total amount of premiums for the i-th class accumulated up to time $t \ge 0$, denoted by $C_i(t)$, is a nonnegative and nondecreasing stochastic process with $C_i(0) = 0$ and $C_i(t) < \infty$ almost surely (a.s.) for every $0 \le t < \infty$;

A4. for the i-th class of business, the diffusion process $\{B_i(t), t \ge 0\}$, as a perturbed term, is a standard Brownian motion with volatility parameter $\sigma_i \ge 0$, and independent of $\{X_k^{(i)}, k \ge 1\}$, $\{N_i(t), t \ge 0\}$

* Corresponding author.
E-mail address: qwgao0123@gmail.com (Q. Gao).

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and $\{C_i(t), t \geq 0\}$. In practice, the perturbed term can be interpreted as an additional uncertainty of the insurer's aggregate claims or its premium income.

Denote the arrival times of successive claims by $\tau_k^{(i)}$, $k \geq 1$, and then $\theta_1^{(i)} = \tau_1^{(i)}$, $\theta_k^{(i)} = \tau_k^{(i)} - \tau_{k-1}^{(i)}$, $k \geq 2$, are the claim inter-arrival times for the i-th class of business, with common distribution G_i, $i = 1, 2$, respectively. Define $\Lambda_i = \{t : EN_i(t) > 0\}$ and $\underline{t}^{(i)} = \inf\{t : EN_i(t) > 0\}$, $i = 1, 2$. Clearly, for $i = 1, 2$, $\Lambda_i = [\underline{t}^{(i)}, \infty]$ if $P(\tau_1^{(i)} = \underline{t}^{(i)}) > 0$, or $\Lambda_i = (\underline{t}^{(i)}, \infty]$ if $P(\tau_1^{(i)} = \underline{t}^{(i)}) = 0$. Put $\Lambda = \Lambda_1 \cap \Lambda_2$.

Let $r \geq 0$ be the constant force of interest, and $x_i \geq 0$ be the initial reserve for the i-th class of business, $i = 1, 2$. Hence, up to time $t \geq 0$, the total reserve for the i-th class, $i = 1, 2$, satisfies

$$U_i(x_i, t) = x_i e^{rt} + \int_{0-}^{t} e^{r(t-s)} dC_i(s) - \sum_{k=1}^{\infty} X_k^{(i)} e^{r(t-\tau_k^{(i)})} \mathbf{1}_{\{\tau_k^{(i)} \leq t\}} + \sigma_i \int_{0-}^{t} e^{r(t-s)} dB_i(s), \quad t \geq 0,$$

and the insurer's total reserve of the two classes of insurance business is

$$U_r(t) = \sum_{i=1}^{2} U_i(x_i, t), \quad t \geq 0, \tag{1.1}$$

where $\mathbf{1}_A$ is the indicator function of an event A. By assumption **A3**, one can easily see that for $i = 1, 2$,

$$0 \leq \widetilde{C}_i(t) = \int_{0-}^{t} e^{-rs} dC_i(s) < \infty \quad \text{a.s., for any } 0 < t < \infty, \tag{1.2}$$

where $\widetilde{C}_i(t)$ is the total discounted value of premiums accumulated up to time $t > 0$ for the i-th class of business.

Now we define three types of ruin times for the diffusion-perturbed bidimensional risk model (1.1) as follows: the first time when both $U_1(x_1, t)$ and $U_2(x_2, t)$ become negative is defined by

$$\tau_{\max}(x_1, x_2) = \inf\bigl\{t : \max\bigl\{U_1(x_1, t), U_2(x_2, t)\bigr\} < 0 \bigm| U_i(x_i, 0) = x_i, i = 1, 2\bigr\};$$

the first time when either $U_1(x_1, t)$ or $U_2(x_2, t)$ becomes negative is

$$\tau_{\min}(x_1, x_2) = \inf\bigl\{t : \min\bigl\{U_1(x_1, t), U_2(x_2, t)\bigr\} < 0 \bigm| U_i(x_i, 0) = x_i, i = 1, 2\bigr\};$$

and the first time when the sum $U_r(t)$ of $U_1(x_1, t)$ and $U_2(x_2, t)$ becomes negative is

$$\tau_{\text{sum}}(x_1, x_2) = \inf\bigl\{t : U_r(t) < 0 \bigm| U_i(x_i, 0) = x_i, i = 1, 2\bigr\}.$$

Then the corresponding ruin probabilities with a finite time $t > 0$ are respectively defined by

$$\psi_{\max}(x_1, x_2, t) = P\bigl(\tau_{\max}(x_1, x_2) \leq t\bigr) = P\left(\bigcap_{i=1}^{2} \bigl\{U_i(x_i, s) < 0\bigr\} \text{ for some } 0 \leq s \leq t\right), \tag{1.3}$$

$$\psi_{\min}(x_1, x_2, t) = P\bigl(\tau_{\min}(x_1, x_2) \leq t\bigr) = P\left(\bigcup_{i=1}^{2} \bigl\{U_i(x_i, s) < 0\bigr\} \text{ for some } 0 \leq s \leq t\right), \tag{1.4}$$

and