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On the uniqueness and analyticity of solutions in micropolar thermoviscoelasticity

Antonio Magaña *, Ramón Quintanilla

Dept. Matemática Aplicada 2, UPC, C. Colón 11, 08222 Terrassa, Barcelona, Spain

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ABSTRACT

This paper deals with the linear theory of isotropic micropolar thermoviscoelastic materials. When the dissipation is positive definite, we present two uniqueness theorems. The first one requires the extra assumption that some coupling terms vanish; in this case, the instability of solutions is also proved. When the internal energy and the dissipation are both positive definite, we prove the well-posedness of the problem and the analyticity of the solutions. Exponential decay and impossibility of localization are corollaries of the analyticity.

1. Introduction

A great effort has been made in the last years to understand the behavior of the so-called “non-classical materials”. Solids with voids, mixtures of materials or non-simple materials are examples of them. Some mathematical and mechanical studies about these materials can be found in the book of Ieşan [11].

It can be said that the first analysis concerning the time decay properties of these alternative solids was proposed by Quintanilla [26]. The author proved the slow decay of the solutions with respect to the time for elastic–porous materials when the only dissipation mechanism is the porous dissipation. After that, a lot of works were intended to clarify the behavior of the solutions (exponential decay, slow decay, impossibility of localization and/or analyticity) for solids with voids [4,5,9,10,13,14,16–18,20,19,21–24,29], for non-simple materials [8,25] or for mixtures of elastic solids [1–3,27,28]. Nevertheless, no attention has been paid up to now to micropolar elastic solids. We believe that these kind of properties are a relevant issue to clarify in order to understand better the thermomechanical behavior of these materials.

The origin of the rational theories about polar continua is attributed to E. and F. Cosserat (see [11] or [7]) at the beginning of the twentieth century. In the sixties, other contributions on this field were done: we want to highlight the work of Eringen [6], among others. Nowadays, these materials are a subclass of the micromorphic materials. Metals, polymers, rocks, wood, ceramics, soils, biological materials or pressed powders are typical examples of them.

In this work we focus on the analysis of the qualitative properties of the isotropic micropolar thermoviscoelastic materials. That is, materials that, apart from the usual macroscopic movements, allow its material points to rotate. We consider thermal effects as well as viscosity effects at the macroscopic and microscopic levels. We have two main purposes. First, we will suppose that the dissipation is positive definite. In this case we will see the uniqueness of the solutions and its instability when the internal energy is not necessarily positive definite. Second, if the dissipation and the internal energy are both positive definite, we will obtain the analyticity of the solutions. This is an important fact: analyticity implies that the solutions are very regular. That means that the orbits are analytic functions in the time variable and that the solution can be obtained from the derivatives in any point. Two consequences of this fact are the exponential decay and the impossibility

* Corresponding author.

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of localization of the solutions. In particular, the exponential stability tells us that the perturbations are damped in a very fast way and, from an empirical point of view, they will be imperceptible after a small period of time.

As we said above, the study of the qualitative properties of the micropolar elastic solids has got little attention. Our aim is to improve this circumstance. We take the situation where the more number of dissipation mechanisms can appear. We think that this could be a good beginning because this situation determines when the solutions will be more regular possible. The analysis of the solutions when less dissipation mechanisms are present could be the aim of future works.

The structure of the paper is the following. In Section 2 we recall the basic equations with the assumptions that we need to set down the problem. In Section 3, we present uniqueness and instability results for the solutions using the logarithmic convexity argument for a particular (but quite general) case. In Section 4, uniqueness is proved working with the general system of equations. The existence of solution is proved in Section 5 making use of the linear operators semigroup theory. Finally, in Section 6, we prove the analyticity of the solutions, that, among other properties, shows that the solutions are exponentially stable and also the impossibility of localization.

2. Basic equations

Let us consider a homogeneous isotropic tridimensional micropolar viscoelastic body which occupies a three-dimensional domain Γ with a boundary, $\partial\Gamma$, smooth enough to apply the divergence theorem. We consider the strain measures e_{ij} and κ_{ij} which are defined by

$$e_{ij} = u_{j,i} + \epsilon_{jik}\phi_k, \qquad \kappa_{ij} = \phi_{j,i}, \tag{2.1}$$

where u_i are the components of the displacement vector, ϕ_i are the components of the microrotation and ϵ_{jik} is the alternating symbol. In this paper we assume that the stress tensor t_{ij}, the microstress m_{ij}, the entropy η and the heat flux vector q_i are related to the strain measures e_{ij} and κ_{ij} and also to the temperature T and the gradient of the temperature by means of the constitutive equations

$$\begin{aligned}
t_{ij} &= \lambda e_{rr}\delta_{ij} + (\mu+\sigma)e_{ij} + \mu e_{ji} + \lambda_v \dot{e}_{rr}\delta_{ij} + (\mu_v+\sigma_v)\dot{e}_{ij} + \mu_v \dot{e}_{ji} - bT\delta_{ij},\\
m_{ij} &= \alpha\kappa_{rr}\delta_{ij} + \beta\kappa_{ji} + \gamma\kappa_{ij} + \alpha_v\dot{\kappa}_{rr} + \beta_v\dot{\kappa}_{ji} + \gamma_v\dot{\kappa}_{ij} + b^*\epsilon_{ijr}T_{,r},\\
\rho\eta &= be_{rr} + aT,\\
q_i &= kT_{,i} + k^*\epsilon_{irs}\dot{\kappa}_{rs}.
\end{aligned}$$

Here λ, μ, σ, λ_v, μ_v,σ_v, b, b^*, α, β, γ, α_v, β_v, γ_v, ρ, a, k and k^* are the constitutive coefficients and δ_{ij} the Kronecker delta.

The evolution equations are

$$\begin{aligned}
&t_{ji,j} + \rho F_i^{(1)} = \rho\ddot{u}_i,\\
&\epsilon_{ijk}t_{jk} + m_{ji,j} + JF_i^{(2)} = J\ddot{\phi}_i,\\
&\rho T_0\dot{\eta} = q_{i,i} + \rho r,
\end{aligned}$$

where ρ and J are positive constants whose physical meaning is well known and $F_i^{(k)}$ and r are the supply terms.

To have a well determined problem we need to impose boundary and initial conditions. Therefore, we will assume Dirichlet boundary conditions

$$u_i(\boldsymbol{x},t) = \bar{u}_i, \qquad \phi_i(\boldsymbol{x},t) = \bar{\phi}_i, \qquad T(\boldsymbol{x},t) = \bar{T}, \quad \boldsymbol{x}\in\partial\Gamma, \tag{2.2}$$

and the following initial conditions

$$\begin{aligned}
&u_i(\boldsymbol{x},0) = u_i^0(\boldsymbol{x}), \qquad \dot{u}_i(\boldsymbol{x},0) = v_i^0(\boldsymbol{x}), \qquad \phi_i(\boldsymbol{x},0) = \phi_i^0(\boldsymbol{x}), \qquad \dot{\phi}_i(\boldsymbol{x},0) = \varphi_i^0(\boldsymbol{x}),\\
&T(\boldsymbol{x},0) = T^0(\boldsymbol{x}), \quad \boldsymbol{x}\in\Gamma.
\end{aligned} \tag{2.3}$$

As the material is isotropic the system of the field equations is given by

$$\begin{aligned}
&(\mu+\sigma)\Delta u_i + (\lambda+\mu)u_{r,ri} + \sigma\epsilon_{irs}\phi_{s,r} + (\mu_v+\sigma_v)\Delta\dot{u}_i + (\lambda_v+\mu_v)\dot{u}_{r,ri} + \sigma_v\epsilon_{irs}\dot{\phi}_{s,r} - bT_{,i} + \rho F_i^{(1)} = \rho\ddot{u}_i,\\
&\gamma\Delta\phi_i + b^*\epsilon_{ijk}T_{,kj} + (\alpha+\beta)\phi_{r,ri} + \sigma\epsilon_{irs}u_{s,r} - 2\sigma\phi_i + \gamma_v\Delta\dot{\phi}_i\\
&\quad + (\alpha_v+\beta_v)\dot{\phi}_{r,ri} + \sigma_v\epsilon_{irs}\dot{u}_{s,r} - 2\sigma_v\dot{\phi}_i + JF_i^{(2)} = J\ddot{\phi}_i,\\
&T_0(b\dot{u}_{i,i} + a\dot{T}) = k\Delta T + k^*\epsilon_{irs}\dot{\kappa}_{rs,i} + \rho r.
\end{aligned} \tag{2.4}$$

Here Δ means the Laplace operator.

As usual in this paper we assume that the coefficients satisfy $\rho > 0$, $J > 0$, $a > 0$ and $k > 0$.