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Global existence and asymptotic behavior of solutions for thermodiffusion equations

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ABSTRACT

In this paper, we consider the initial–boundary value problems for the 1D thermodiffusion equations in a bounded region. Using the semigroup approaches and the multiplier methods, we obtain the global existence and asymptotic behavior of solutions for homogeneous, nonhomogeneous and semilinear thermodiffusion equations subject to various boundary conditions, respectively.

1. Introduction

We consider the initial–boundary value problems for the 1D thermodiffusion equations in a bounded region $\Omega = (0, 1)$. The thermodiffusion equations describe the process of thermodiffusion in a solid body. Such a process is described by the following system of equations (see, e.g., [9,10]):

$$\rho u_{tt} - (\lambda + 2\mu)u_{xx} + \gamma_1\theta_{1x} + \gamma_2\theta_{2x} = f, \quad in\ (0, 1) \times (0, +\infty), \tag{1.1}$$

$$c\theta_{1t} - k\theta_{1xx} + \gamma_1 u_{tx} + d\theta_{2t} = g, \quad in\ (0, 1) \times (0, +\infty), \tag{1.2}$$

$$n\theta_{2t} - D\theta_{2xx} + \gamma_2 u_{tx} + d\theta_{1t} = h, \quad in\ (0, 1) \times (0, +\infty), \tag{1.3}$$

together with the initial conditions

$$u(x, 0) = u_0(x), \qquad u_t(x, 0) = u_1(x), \qquad \theta_1(x, 0) = \theta_{10}(x), \qquad \theta_2(x, 0) = \theta_{20}(x), \tag{1.4}$$

and one of the following boundary conditions:

$$u(x, t)|_{x=0,1} = 0, \qquad \theta_1(x, t)|_{x=0,1} = 0, \qquad \theta_2(x, t)|_{x=0,1} = 0, \tag{1.5}$$

$$u(x, t)|_{x=0,1} = 0, \qquad \theta_{1x}(x, t)|_{x=0,1} = 0, \qquad \theta_{2x}(x, t)|_{x=0,1} = 0, \tag{1.6}$$

$$u(x, t)|_{x=0,1} = 0, \qquad \theta_1(x, t)|_{x=0,1} = 0, \qquad \theta_{2x}(x, t)|_{x=0,1} = 0, \tag{1.7}$$

$$u(x, t)|_{x=0,1} = 0, \qquad \theta_{1x}(x, t)|_{x=0,1} = 0, \qquad \theta_2(x, t)|_{x=0,1} = 0, \tag{1.8}$$

$$u_x(x, t)|_{x=0,1} = 0, \qquad \theta_1(x, t)|_{x=0,1} = 0, \qquad \theta_2(x, t)|_{x=0,1} = 0, \tag{1.9}$$

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where u, θ_1, and θ_2 are the displacement, temperature, and chemical potential as independent fields. These fields are dependent on the space variable x and the time variable t. Functions f, g and h are known functions specified later on.

Here, we denote by λ, μ the material coefficients, ρ the density, γ_1, γ_2 the coefficients of thermal and diffusion dilatation, k the coefficient of thermal conductivity, D the coefficient of thermal conductivity, and n, c, d the coefficients of thermodiffusion. All the above coefficients are positive constants and satisfy

$$nc - d^2 > 0. \tag{1.10}$$

The condition (1.10) implies that (1.1)–(1.3) is a hyperbolic–parabolic system and similar with the one-dimensional linear thermoelastic system (see, e.g., [5–8]). The total energy for these problems is given by

$$E(t) = \int_0^1 [(\lambda + 2\mu)u_x^2 + \rho u_t^2 + c\theta_1^2 + n\theta_2^2 + 2d\theta_1\theta_2]dx. \tag{1.11}$$

We shall first recall some related results in the literature. The thermodiffusion equations (1.1)–(1.3) were first given by Nowacki [9,10] in the homogeneous form (i.e., $f = g = h \equiv 0$). Nowacki [10], Podstrigach [11] and Fichera [1] investigated the initial–boundary value problem for the linear homogeneous system by the method of integral transformations and integral equations. Gawinecki [3] proved the existence, uniqueness and regularity of solutions to the initial–boundary value problem for the linear system of thermodiffusion in a solid body. Using the Fourier transform, the matrix of fundamental solutions (cf., [2,4]) was constructed for three cases: for the linear system of thermodiffusion, in the quasi-static case of the thermal stresses theory, for the whole system of equations. Recently, Szymaniec [12] proved L^p–L^q time decay estimates for the solution of the linear thermodiffusion system. Using the results in [12], Szymaniec [13] then obtained the global existence and uniqueness of solutions to the Cauchy problem of nonlinear thermodiffusion equations in a solid body.

However, the global existence and asymptotic behavior of solutions for homogeneous, nonhomogeneous and semilinear thermodiffusion equations subject to various boundary conditions are still open. In this paper, we shall use the semigroup approaches and the multiplier methods to establish the global existence and exponential stability of solutions for the associated problems.

The notation in this paper will be as follows:

L^p, $W^{m,p}$, $1 \le p \le +\infty$, $m \in N$, $H^1 = W^{1,2}$, $H_0^1 = W_0^{1,2}$, $H^2 = W^{2,2}$, denote the usual (Sobolev) spaces on (0, 1). In addition, $\|\cdot\|_B$ denotes the norm in the space B; we also put $\|\cdot\| = \|\cdot\|_{L^2}$. We denote by $C^k(I, B)$, $k \in N_0$, the space of k-times continuously differentiable functions from $I \subseteq R$ into a Banach space B, and likewise by $L^p(I, B)$, $1 \le p \le +\infty$, the corresponding Lebesgue spaces. Subscripts t and x denote the partial derivatives with respect to t and x, respectively.

For readers' convenience, we include here some lemmas on the semigroup theory.

Lemma 1.1 (*[8]*). *Let A be a linear operator with dense domain $D(A)$ in a Hilbert space H. If A is dissipative and $0 \in \varrho(A)$, the resolvent set of A, then A is the infinitesimal generator of a C_0-semigroup of contractions on H.*

Lemma 1.2 (*[14,8]*). *Let $S(t) = e^{At}$ be a C_0-semigroup of contractions on a Hilbert space. Then $S(t)$ is exponentially stable if and only if*

$$\varrho(A) \supseteq \{i\beta, \beta \in R\} \equiv iR \tag{1.12}$$

and

$$\overline{\lim_{|\beta|\to+\infty}} \|(i\beta I - A)^{-1}\| < +\infty \tag{1.13}$$

hold.

Consider the following initial value problem for the abstract first-order equation:

$$\begin{cases} \dfrac{dy}{dt} + Ay = K, \\ y(0) = y_0, \end{cases} \tag{1.14}$$

where A is the infinitesimal generator of a C_0-semigroup of contractions defined in a dense subset $D(A)$ of a Banach space B. We have the following lemmas.

Lemma 1.3 (*[15] Homogeneous Case*). *Let $K \equiv 0$, suppose that $y_0 \in D(A)$. Then problem (1.14) has a unique classical solution $y(t)$ such that*

$$y(t) \in C^1([0, +\infty), B) \cap C([0, +\infty), D(A)).$$

Lemma 1.4 (*[15] Nonhomogeneous Case*). *Let $K = K(x, t)$, suppose that $y_0 \in D(A)$ and $K(t) \in C^1([0, +\infty), B)$. Then problem (1.14) admits a unique global classical solution $y(t)$ such that*

$$y(t) \in C^1([0, +\infty), B) \cap C([0, +\infty), D(A)).$$