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Dimensions of random average conformal repellers

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ABSTRACT

This paper defines random average conformal repellers which are a generalization of random conformal repellers. Using the subadditive thermodynamic formalism, the Hausdorff dimension of random average conformal repellers is obtained as the root of the random pressure function.

1. Introduction

It is well known that many self-similar sets, or fractals, can be realized as invariant sets of smooth expanding maps. The Hausdorff dimension and the box dimension of these sets play an important role in various areas of dynamical systems, and have attracted a great deal of attention; see [12,18]. Computing these fractal invariants is usually difficult, because they depend on the microscopic structure of the set.

Researchers in dynamical systems and dimension theory have already created a few effective ways to calculate the dimension of sets invariant under nonlinear maps in dimension larger than one. So far the theory has been developed only to full satisfaction in the case of conformal dynamical systems (both invertible and non-invertible). Bowen first related the dimension of repellers to the zero of the topological pressure function. In [6], he proved that the root of Bowen's equation was the Hausdorff dimension of a conformal repeller. Recently, different versions of the topological pressure have become a useful tool in calculating the Hausdorff dimension of a repeller.

Gatzouras and Peres [14] obtained the Hausdorff dimension of a C^1 conformal repeller as the root of Bowen's equation. Let us mention some recent results on the dimensions of non-conformal repellers. Zhang [22] investigated the Hausdorff dimension of C^1 non-conformal repellers. Barreira [3] considered the box dimension of non-conformal repellers under the additional assumptions that the map is $C^{1+\alpha}$ and α-bunched. In [2], the authors proved that the upper bound of the Hausdorff dimension for C^1 non-conformal repellers obtained in [3,13,22] were the same and it was the unique root of Bowen's equation for the subadditive topological pressure which was studied in [8]. Particularly, they introduced the notion of C^1 average conformal repellers which were a generalization of quasi-conformal and asymptotically conformal repellers in [4,18], and they proved that the Hausdorff dimension and the box dimension of average conformal repellers were the unique root of Bowen's equation for the subadditive topological pressure. Furthermore, authors [24] proved that the Hausdorff dimension of average conformal repellers was stable under random perturbations; see [9] for the dimension estimates of an ergodic measure supported on average conformal repellers.

For random repellers, Kifer [15] proved that the Hausdorff dimension of a measurable C^2 random conformal repeller was the root of the random Bowen's equation. Bogenschütz and Ochs [5] generalized this result to almost-conformal repellers, where the upper and lower bounds of dimension estimates for such repellers were obtained as the roots of random

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Bowen's equations. Crauel and Flandoni [11] considered the Hausdorff dimension of random repellers for certain classes of transformation. Rugh [19] considered random C^1 conformal repellers under the assumption of uniform equi-expansion and equi-mixing, and obtained that the Hausdorff and box dimensions were bounded respectively with the unique zeros of a lower and an upper conformal pressure.

Motivated by the work in [2], this paper introduces the notion of random average conformal repellers. Using the random subadditive topological pressure defined in [23], we prove that the Hausdorff dimension of a random average conformal repeller is the unique root of Bowen's equation for random subadditive topological pressure. At the same time, we introduce random superadditive topological pressure and prove that random subadditive and superadditive topological pressure are the same for special potentials.

In this paper, we use the subadditive topological pressure to obtain the Hausdorff dimension of random $C^{1+\alpha}$ average conformal repellers. Under the assumption of uniform equi-expansion and equi-mixing, we also can obtain that the Hausdorff and box dimensions of random C^1 average conformal repellers and the lower and upper bound dimension estimates of random C^1 non-conformal repellers. We will consider these in the further-coming papers.

The paper is organized as follows. In Section 2, we recall the variational principle of random subadditive topological pressure. In Section 3, we give some preliminary results of random average conformal repellers. In Section 4, we discuss the superadditive thermodynamic formalism and variational principle of topological pressure for a special superadditive potential. In Section 5, we give the proof of the main result.

In the following, we recall some basic definitions and notations in random dynamic systems.

Let ϑ be an ergodic invertible transformation of a Lebesgue space $(\Omega, \mathcal{W}, \mathbb{P})$. Further, let E be a compact bundle over Ω with fibers in a Polish space M, that is say, $E \subset \Omega \times M$ is a measurable set such that all ω-sections $E_\omega = \{x \in M \mid (\omega, x) \in E\}$ are compact. If $\mathcal{K}$ denotes the collection of all compact subsets of M endowed with the Hausdorff topology, this is equivalent to saying that $\mathcal{K}$-valued multifunction $\omega \mapsto E_\omega$ is measurable. Let T be a continuous bundle random dynamic system (RDS for short) over ϑ, i.e. $T_\omega : E_\omega \to E_{\vartheta\omega}$ is continuous $\mathbb{P}$-a.s. Here and in what follows we think of E_ω being equipped with the trace topology, i.e. an open set $A \subset E_\omega$ is of the form $A = B \cap E_\omega$ with some open set $B \subset M$. And we call E is T-invariant if $T_\omega E_\omega = E_{\vartheta\omega}$ $\mathbb{P}$-a.s. The map $\Theta : E \to E$ is defined by $\Theta(\omega, x) = (\vartheta\omega, T_\omega x)$, and we call it the *skew product transformation*.

1.1. Random average conformal repeller

Let M be an m-dimensional C^∞ compact Riemannian manifold with a Riemannian metric d. Consider a measurable family $T = \{T_\omega : M \to M \mid \omega \in \Omega\}$ of $C^{1+\alpha}$ maps, i.e., $(\omega, x) \mapsto T_\omega(x)$ is assumed to be measurable. Let E be a measurable T-invariant set. And let $\|DT_\omega(x)\|$ be the norm of $DT_\omega(x)$, and $m(DT_\omega(x))$ the minimum norm of $DT_\omega(x)$, i.e., $m(DT_\omega(x)) = \inf_{0\neq u\in T_xM} \frac{\|DT_\omega(x)(u)\|}{\|u\|}$. Since $T_\omega \in C^{1+\alpha}$, there exists a positive random variable $r(\omega)$ with $\log r(\omega) \in L^1(\mathbb{P})$ such that for $\mathbb{P} - a.e\, \omega, x, y \in E_\omega$ and $d(x, y) \le r(\omega)$, it has $||DT_\omega(x)| - |DT_\omega(y)|| \le C(\omega)|x - y|^\alpha$ for some positive random variable $C(\omega)$, where $|DT_\omega(x)|$ denotes the Jacobian of $DT_\omega(x) : T_xM \to T_{T_\omega x}M$. We assume $\log^+ C(\omega) \in L^1(\mathbb{P})$ and $\log \|DT_\omega(x)\|,\ \log m(DT_\omega(x)) \in L^1_E(\Omega, C(M))$ (see Section 2 for the precise definition of $L^1_E(\Omega, C(M))$). Therefore for $\mathbb{P}$-a.e. $\omega, x, y \in E_\omega$ and $d(x, y) \le r(\omega)$, we have

$$m(DT_\omega(x)) - C(\omega)d(x, y)^\alpha \le \frac{d(T_\omega x, T_\omega y)}{d(x, y)} \le \|DT_\omega(x)\| + C(\omega)d(x, y)^\alpha \tag{1.1}$$

Let $\mathcal{M}_{\mathbb{P}}(E, T)$ denote the space of Θ-invariant probability measures with marginal $\mathbb{P}$ on Ω, and $\mathcal{E}_{\mathbb{P}}(E, T)$ is a subset of $\mathcal{M}_{\mathbb{P}}(E, T)$ with ergodic invariant measures. By the Oseledec multiplicative ergodic theorem, for any $\mu \in \mathcal{E}_{\mathbb{P}}(E, T)$, we can define Lyapunov exponents $\lambda_1(\mu) \le \lambda_2(\mu) \le \cdots \le \lambda_m(\mu),\ m = \dim M$.

Definition 1.1. A measurable invariant set E is called a *random average conformal repeller* if for any $\mu \in \mathcal{E}_{\mathbb{P}}(E, T)$, $\lambda_1(\mu) = \lambda_2(\mu) = \cdots = \lambda_m(\mu) > 0$.

Remark 1. E is a random average conformal repeller, then T is *random uniformly expanding* on E, namely, there exist a positive valued random variable $K(\omega)$ and a positive constant λ such that for $\mathbb{P}$-almost all $\omega \in \Omega$ and every $x \in E_\omega$ we have

$$\|DT^n_\omega(x)v\| \ge K(\omega)e^{\lambda n}\|v\|, \quad \forall v \in T_xM,\ \forall n \in \mathbb{N} \tag{1.2}$$

where $K(\omega)$ satisfying $\lim_{n\to\infty} \frac{1}{n} \log K(\vartheta^n\omega) = 0,\ \mathbb{P}$-a.s., see [10] for a detailed proof.

The main result in this paper is the following theorem which was stated as Theorem 5.1 in [23]. The definition of subadditive and superadditive topological pressure will be defined lately.

Theorem 1.1. *Suppose T is a $C^{1+\alpha}$ RDS, and E is a random average conformal repeller of T, then for $\mathbb{P}$ almost every ω we have*

$$\dim_{\mathrm{H}}(E_\omega) = \bar{t},$$

where $\bar{t}$ is the unique root of equation $\pi^*_T(-t\Phi) = 0$, and it is also the unique root of equation $\pi_T(-t\Psi) = 0$, where $-t\Phi = \{-t \log \|DT^n_\omega(x)\|\}$ and $-t\Psi = \{-t \log m(DT^n_\omega(x))\}$.