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On the theory of thermoelasticity with microtemperatures

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ABSTRACT

This paper studies the linear theory of thermoelastic materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures. We discuss uniqueness results within the context of the dynamic boundary value problems under very mild assumptions upon the thermoelastic profile. We also establish the relations describing the partition of the total energy in terms of the Cesàro means of various types of component energies.

1. Introduction

This paper is concerned with the linear theory of thermoelasticity with microtemperatures as developed by Ieşan and Quintanilla [1]. Such a theory takes into consideration the microstructure of the body and, consequently, each microelement possesses a microtemperature. The theory of elastic materials with microstructures, goes back to the book of E. and F. Cosserat [2]. After that, the theory of materials with microstructures became a subject of intensive study in the literature (see, for example, the articles [3–7] and the books [8–11]).

Grot [12] was the first to take into consideration the inner structure of a body in order to develop a thermodynamic theory for thermoelastic materials where microelements, in addition to classic microdeformations, possess microtemperatures. The Clausius–Duhem inequality is modified to include microtemperatures and the first-order moment of the energy equations are added to the common balance laws of a micromorphic continuum. Riha [13] developed a further study concerning heat conduction in thermoelastic materials with inner structure. It is shown that the experimental data for the silicone rubber containing spherical aluminum particles and for human blood are conform closely to the predicted theoretical model of thermoelasticity with microtemperatures.

The theory of thermoelasticity with microtemperatures has been further investigated in various papers. Thus, Riha [14,15] studies a theory of heat conducting micropolar fluids with microtemperatures, while Ieşan [16,17] develops a linear theory for elastic materials with inner structure whose particles, in addition to the classical displacement and temperature, possess microtemperatures and can stretch and contract independently of their translations. Recently, Ieşan and Quintanilla [18] have established a linear theory of thermoelastic solids with microstructures and microtemperatures which permits the transmission of heat as thermal waves at finite speed and Ieşan [19] derives a linear theory of microstretch elastic solids with microtemperatures (see also the recent book by Straughan [20] and the papers cited therein).

In [1] Ieşan and Quintanilla consider the simplest thermomechanical theory of elastic materials that takes into account the microtemperature variables and then they establish some basic results concerning the uniqueness, existence and asymptotic

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behavior of dynamic solutions and some equilibrium specific problems. As regards the uniqueness result, it is established by assuming that the strain energy and the dissipation energy are positive semi-definite quadratic forms and the specific heat and the specific coefficient of the first moment of energy vector are strictly positive.

Since then the theory of thermoelasticity with microtemperatures has attracted much attention in connection with the study of the basic qualitative properties of solutions to the problems relating to various thermomechanical situations. Thus, Svanadze [21,22], Scalia and Svanadze [23,24] and Scalia et al. [25] study the fundamental solutions and proves some existence and uniqueness theorems for equilibrium solutions and steady state vibrations by means of the potential method, while Ieşan and Scalia [26] consider the plane strain in a homogeneous and isotropic body with microtemperatures. The behavior of shock waves and higher-order discontinuities which propagate in a thermoelastic body with inner structure and microtemperatures are studied by Ieşan [27] and the propagation of singularities of solutions to the Cauchy problem of a semilinear thermoelastic system with microtemperatures in one space variable is studied by Yang and Huang in [28]. Some basic theorems are established by Aouadi [29] and Svanadze and Tracina [30] in the linear theory of microstretch thermoelasticity for isotropic solids with microtemperatures. Finally, Ciarletta et al. [31] investigate a model for a rigid heat conductor which allows for variation of thermal properties at a microstructure level and they examine how the solution depends on changes in coupling coefficients between the macrothermal and microthermal levels.

In this paper we consider the linear theory of thermoelasticity with microtemperatures as developed in [1]. The uniqueness problem for solutions of the initial boundary value problems of the theory in concern is treated by means of the Lagrange identities method [32] without any sign-defined assumption upon the elastic energy and under very mild assumptions upon the other thermoelastic coefficients, including those describing the dissipation energy. This is possible because of the special coupling between the differential equations for the temperature variation and that for the microtemperature field. Such a treatment of the uniqueness problem is more complete with respect to that given in [1,33]. Furthermore, we show how the Lagrange identities method can be used to establish the asymptotic partition of total energy associated with the solutions of the initial–boundary value problems of the theory in concern. We materialize the idea of partition in terms of the asymptotic behavior in the Cesarò sense of various component energies. Thus, we prove that the Cesarò mean of the thermal energy due to the microtemperature fields goes to zero as time tends to infinity, while the Cesarò mean of the thermal energy due to the temperature variation tends to a constant when time tends to infinity. The results established in the present paper concerning the asymptotic partition of energy generalize those developed by Chiriţă [34].

2. Basic equations

Throughout this section B is a bounded regular region of the three-dimensional Euclidean space. We let ∂B denote the boundary of B, and designate by $\mathbf{n}$ the outward unit normal on ∂B. We assume that the body occupying B is a linearly elastic material which possesses thermal variation at a microstructure level. The body is referred to a fixed system of rectangular Cartesian axes Ox_i $(i = 1, 2, 3)$. Throughout this paper Latin indices have the range 1, 2, 3, Greek indices have the range 1, 2 and the usual summation convention is employed. We use a subscript preceded by a comma for partial differentiation with respect to the corresponding coordinate and a superposed dot denotes partial differentiation with respect to time.

The temperature at a point $\mathbf{x}$ of the body depends on a temperature $\theta(\mathbf{x}, t)$, which may be thought of as an averaged temperature at $\mathbf{x}$, and three microtemperatures $w_i(\mathbf{x}, t)$ which contribute to the thermal microstructure of the material. The deformation of a body can be described by means of three, namely, the displacement vector field $\mathbf{u}$, the microtemperature vector field $\mathbf{w}$ and the temperature variation T, measured from the constant absolute temperature $T_0(>0)$, over $B \times [0, \infty)$.

Within the framework of the linear theory developed by Ieşan and Quintanilla [1], the constitutive equations for a homogeneous and isotropic thermoelastic solid with microtemperatures are

$$\begin{aligned}
&t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} - \beta T\delta_{ij},\\
&\varrho\eta = \beta e_{rr} + aT,\\
&\varrho\varepsilon_i = -bw_i,\\
&q_i = kT_{,i} + \kappa_1 w_i,\\
&Q_i = (\kappa_1 - \kappa_2)\, w_i + (k - \kappa_3)\, T_{,i},\\
&q_{ij} = -\kappa_4 w_{r,r}\delta_{ij} - \kappa_5 w_{i,j} - \kappa_6 w_{j,i},
\end{aligned} \tag{2.1}$$

where

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2.2}$$

Here, t_{ij} are the components of the stress tensor, ϱ is the reference mass density, η is the entropy per unit mass, ε_i are the components of the first moment of energy vector, q_i are the components of the heat flux vector, Q_i are the components of the mean heat flux vector, q_{ij} are the components of the first heat flux moment vector, e_{ij} are the components of the strain tensor, u_i are the components of the displacement vector, w_i are the components of the microtemperature vector, T is the temperature variation, λ, μ, β, a, b, k and κ_r $(r = 1, 2, \ldots, 6)$ are constant constitutive coefficients and δ_{ij} is the Kronecker delta.