

A fast wavelet block Jacobi method

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ABSTRACT

In this paper, we develop a fast block Jacobi method for linear systems based on discrete wavelet transform (DWT). Traditional wavelet-based methods for linear systems do not fully utilize the sparsity and the multi-level block structure of the transformed matrix after DWT. For the sake of numerical efficiency, we truncate the transformed matrix to be a sparse matrix by letting the small values be zero. To combine the advantages of the direct method and the iterative method, we solve the sub-systems appropriately based on the multi-level block structure of the transformed matrix after DWT. Numerical examples show that the proposed method is very numerically effective.

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1. Introduction

Numerous problems in science and engineering give rise to the linear system

$$\mathbf{Ax} = \mathbf{b}, \quad (1.1)$$

where $\mathbf{A} \in \mathbb{R}^{N \times N}$, $\mathbf{x}, \mathbf{b} \in \mathbb{R}^N$, $\mathbf{x}$ denotes the unknown vector, $\mathbf{b}$ is a given vector and $\mathbb{R}$ denotes the set of real numbers. Usually, there are two types of methods to solve the linear system (1.1), the direct methods and the iterative methods. Direct methods, such as the Gaussian elimination and *LU* factorization method, compute the exact solution after finite steps. However, the computational complexity of the direct method is usually too high for large scale linear systems. The iterative methods, such as the Jacobi method and Gauss–Seidel method, are more suitable for large sparse linear systems, which gain a good approximation of the exact solution [3,12,19].

Wavelet analysis is an important tool in many areas such as image processing, signal analysis and scientific computing [5,7–9,14,16]. The ability to give a sparse representation of data makes it a tool of increasing importance. Discrete wavelet transform (DWT) is the discrete variant of the wavelet transform. A matrix after two-dimensional (2-D) DWT has a special block structure. The upper left block of the transformed matrix corresponds to the approximation coefficients (low frequency information) which keep the most important information. The remaining blocks correspond to details coefficients (high frequency information) which possess small values almost close to zero. The upper left block is a coarse version of the original matrix. For example, we consider the “cameraman” image in Fig. 1(a) as the matrix $\mathbf{A}$. The first and second level wavelet decomposition are presented in Fig. 1(b) and (c). As it can be seen, after DWT, the main information of transformed matrix are concentrated on the upper left block.

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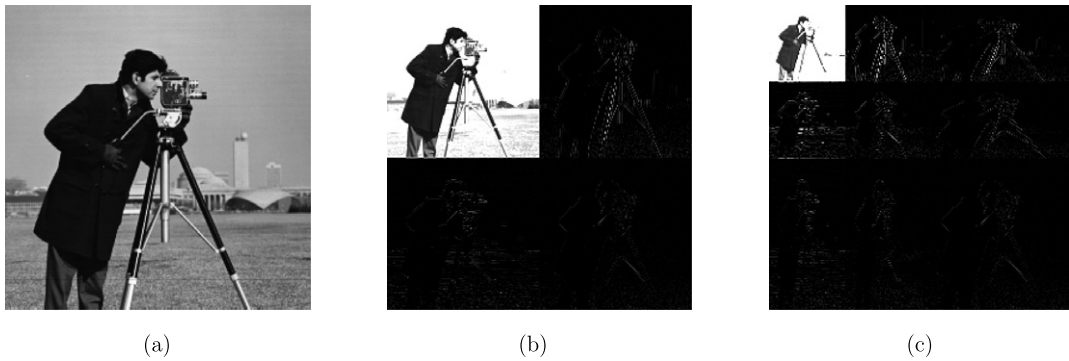


Fig. 1. (a) Original "cameraman" image; (b) The first level wavelet decomposition; (c) The second level wavelet decomposition.

Exploiting such a block structure, several numerical methods have been developed to solve the linear system (1.1) in recent years. For the direct method based on DWT, a multiresolution LU factorization method was proposed based on non-stand DWT in [11]. Using the upper left block as a coarse grid, wavelet algebraic multigrid (WAMG) methods were proposed recently [6,10,13,18,20]. However, the numerical methods mentioned above have not utilized the sparsity of the data after DWT.

In this paper, we develop a fast block Jacobi method based on the special block structure and sparsity of the transformed matrix after DWT. In order to make the transformed matrix sparser, we truncate the small values to be zero. The error between the approximate solution and the exact solution is controlled by the truncation. Furthermore, in order to improve the numerical efficiency, we solve the sub-systems in different ways. We solve the small sub-systems with low frequency information exactly by a direct method, and solve the large sparse sub-systems with high frequency information approximately by an iterative method. Numerical examples show that the proposed method is numerical effective and combines the advantages of the direct method and the iterative method.

This paper is organized as follows. Section 2 gives a short review of discrete wavelet transform. In Section 3, we develop a block Jacobi method based on wavelet truncation, and analyze the convergence. In Section 4, numerical examples are presented to illustrate the efficiency of the proposed method. Finally, in Section 5, we draw our conclusion.

2. Discrete wavelet transform

In this section, we give a short review of the discrete wavelet transform (cf. [15]). In practice, DWT can be understood as the application of a pair of filters: low-pass filters derived from the scaling function and high-pass filters derived from the wavelet function.

Let G be a low-pass filter which is determined by coefficients $\{g_i\}_{i=0}^{L-1}$ and H be a high-pass filter which is determined by coefficients $\{h_i\}_{i=0}^{L-1}$. They are filters of length L and order $L-1$. In this paper, we concentrate on orthogonal wavelets which are the most popular group of wavelets. Then, the filter coefficients must satisfy $h_i = (-1)^i g_{L-1-i}$ for $i = 0, \dots, L-1$ in order to obtain a reversible DWT.

The discrete signal in one dimension (1-D) can be represented by a vector $\mathbf{v}$. Let the length of $\mathbf{v}$ be $N = 2^n$, where n is a positive integer. Then, the one-level DWT over $\mathbf{v}$ can be carried out by convolving $\mathbf{v}$ with filters G and H respectively, getting two sequences of length N . Next, by the elimination of one out of two samples (sub-sampling), two resulting sequences of length $\frac{N}{2}$ are obtained: a sequence $\{d_i\}$ of details coefficients (wavelet coefficients) and a sequence $\{a_i\}$ of coefficients called approximation coefficients. The approximation coefficients keep the most important information of $\mathbf{v}$, whereas most of the elements of the details coefficients are very small. The 1-D DWT over a vector is equivalent to the multiplication of vector by matrix. Let $\mathbf{G} \in \mathbb{R}^{\frac{N}{2} \times N}$ be a matrix defined by coefficients $\{g_i\}_{i=0}^{L-1}$ as follows

$$\mathbf{G} := \begin{bmatrix} g_{L-1} & g_{L-2} & g_{L-3} & \cdots & g_1 & g_0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & g_{L-1} & g_{L-2} & g_{L-3} & \cdots & g_1 & g_0 & \cdots & 0 \\ \vdots & & & & \vdots & & & & & \vdots \\ g_{L-3} & \cdots & g_1 & g_0 & 0 & \cdots & \cdots & 0 & g_{L-1} & g_{L-2} \end{bmatrix}.$$

The matrix $\mathbf{H}$ is defined in the similar way with coefficients $\{h_i\}_{i=0}^{L-1}$. And the coefficients $\{h_i\}_{i=0}^{L-1}$ satisfy $h_i = (-1)^i g_{L-1-i}$ for $i = 0, \dots, L-1$. The transform matrix $\mathbf{W}_n \in \mathbb{R}^{N \times N}$ is defined as follows

$$\mathbf{W}_n := \begin{bmatrix} \mathbf{G} \\ \mathbf{H} \end{bmatrix}. \quad (2.1)$$

The nonzero entries of $\mathbf{G}$ correspond to low-pass filters, and those of $\mathbf{H}$ correspond to high-pass filters. Multiplying vector $\mathbf{v}$ by matrix $\mathbf{W}_n$, we have

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