J. Math. Anal. Appl. 376 (2011) 481–493

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Random attractors for stochastic sine-Gordon lattice systems with multiplicative white noise

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ARTICLE INFO

Article history:
Received 27 January 2010
Available online 18 November 2010
Submitted by Goong Chen

Keywords:
Random attractor
Stochastic sine-Gordon lattice equations
Random asymptotic nullness

ABSTRACT

We study the asymptotic behavior of solutions to the stochastic sine-Gordon lattice equations with multiplicative white noise. We first prove the existence and uniqueness of solutions, and then establish the existence of tempered random bounded absorbing sets and global random attractors.

1. Introduction

Lattice dynamical systems (LDS's) arise in many applications of science and engineering where the spatial structure has a discrete character. The wide range of interest includes metallurgy in material science, where LDS's have been used to model solidification of alloys [5], cellular neutral networks in electrical circuit theory [9,10,20], and some other examples in pattern recognition, image processing, chemical reaction theory, etc. Therefore the research on LDS's has attracted much attention from both mathematicians and engineering scientists. Extensive studies have been made on different aspects of LDS's during the past decades, such as the chaos properties of solutions [7,16], traveling wave solutions [8,20,21], and existence of the global attractors for LDS's [3,4].

Since most of the realistic systems involve random effects which may play an important role as intrinsic phenomena rather than just compensation of defects in deterministic models, stochastic lattice systems (SLDS's) then arise naturally while these uncertainties or noises are taken into account. Many works have been done regarding the existence of global random attractors for LDS's driven by stochastic processes [2,6,14]. Not until recently did researchers start to pay attention to the existence of random attractors for second-order SLDS's [17–19]. However, all the existing work only consider the case of additive noise, no result has been presented on second-order SLDS's with multiplicative noise to date.

The sine-Gordon equation,

$$\frac{\partial^2 u}{\partial t^2} = a\frac{\partial^2 u}{\partial x^2} + b\sin(\lambda u), \tag{1.1}$$

is a nonlinear hyperbolic differential equation which has a wide range of applications in physics, not only in relativistic field theories but in solid-state physics, nonlinear optics, etc. Fan studied the attractors for a damped sine-Gordon equation with multiplicative noise [11,12] on a bounded domain, but not the sine-Gordon lattice systems. In this article we will discuss the random attractors for stochastic sine-Gordon lattice systems with multiplicative white noise. The rest of this paper is organized as follows. In Section 2, we introduce basic concepts concerning random dynamical systems and global random

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0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2010.11.032

attractors. We then discuss the existence of random attractors for stochastic sine-Gordon lattice systems with multiplicative white noise in Section 3. Section 4 offers some closing remarks.

2. Preliminaries

In this section, we present some concepts (from [1,2]) related to a RDS and a random attractor.

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space, $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $\{\theta_t : \Omega \mapsto \Omega,\ t \in \mathbb{R}\}$ be a family of measure preserving transformations such that $(t,\omega) \mapsto \theta_t\omega$ is measurable, $\theta_0 = \mathrm{Id}_\Omega$, $\theta_{t+s} = \theta_t\theta_s$, for all $s, t \in \mathbb{R}$. The space $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ is called a *metric dynamical system*.

In the following, a property holds for a.e. $\omega \in \Omega$ means that there is $\Omega_0 \subset \Omega$ with $\mathbb{P}(\Omega_0) = 1$ and $\theta_t\Omega_0 = \Omega_0$ for $t \in \mathbb{R}$.

Definition 2.1. A continuous random dynamical system on $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ with state space X is a $(\mathcal{B}(\mathbb{R}^+) \times \mathcal{F} \times \mathcal{B}(X), \mathcal{B}(X))$-measurable mapping

$$\varphi : \mathbb{R}^+ \times \Omega \times X \to X, \quad (t, \omega, u) \mapsto \varphi(t, \omega, u)$$

such that the following properties hold

(1) $\varphi(0, \omega, u) = u$ for all $\omega \in \Omega$ and $u \in X$;
(2) $\varphi(t+s, \omega, \cdot) = \varphi(t, \theta_s\omega, \varphi(s, \omega, \cdot))$ for all $s, t \geqslant 0$ and $\omega \in \Omega$;
(3) φ is continuous in t and u.

For given $u \in X$ and $E, F \subset X$, we define

$$d(u, E) = \inf_{v\in E} \|u - v\|_X$$

and

$$d_H(E, F) = \sup_{u\in E} d(u, F).$$

$d_H(E, F)$ is called the *Hausdorff semi-distance* from E to F.

Definition 2.2.

(1) A set-valued mapping $\omega \mapsto D(\omega) : \Omega \to 2^X$ is said to be a *random set* if the mapping $\omega \mapsto d(u, D(\omega))$ is measurable for any $u \in X$.
(2) The mapping $\omega \mapsto D(\omega)$ is called a *random closed* (*compact*) *set*, if $D(\omega)$ is closed (compact) for each $\omega \in \Omega$.
(3) A random set $\omega \mapsto D(\omega)$ is said to be *bounded* if there exist $u_0 \in X$ and a random variable $R(\omega) > 0$ such that
$$D(\omega) \subset \big\{u \in X\colon \|u - u_0\|_X \leqslant R(\omega)\big\} \quad \text{for all } \omega \in \Omega.$$
(4) A random bounded set $\omega \mapsto D(\omega)$ is called *tempered* if for a.e. $\omega \in \Omega$,
$$\lim_{t\to\infty} e^{-\beta t} \sup\big\{\|u\|_X\colon u \in D(\theta_{-t}\omega)\big\} = 0 \quad \text{for all } \beta > 0.$$

Definition 2.3. A random set $\omega \mapsto B(\omega)$ is said to be a *random absorbing set* if for any tempered random set $\omega \mapsto D(\omega)$, there exists $T_D(\omega)$ such that

$$\varphi\big(t, \theta_{-t}\omega, D(\theta_{-t}\omega)\big) \subset B(\omega) \quad \text{for all } t \geqslant T_D(\omega),\ \omega \in \Omega.$$

Definition 2.4. A random set $\omega \mapsto B_1(\omega)$ is said to be a *random attracting set* if for any tempered random set $\omega \mapsto D(\omega)$, we have

$$\lim_{t\to\infty} d_H(\varphi\big(t, \theta_{-t}\omega, D(\theta_{-t}\omega), B_1(\omega)\big) = 0 \quad \text{for all } \omega \in \Omega.$$

Definition 2.5. A random compact set $\omega \mapsto A(\omega)$ is said to be a random attractor if it is an random attracting set and $\varphi(t, \omega, A(\omega)) = A(\theta_t\omega)$ for all $\omega \in \Omega$ and $t \geqslant 0$.

In what follows we use $\mathcal{D}(X)$ to denote the set of all tempered random subsets of X.

Definition 2.6. A RDS $\{\varphi(t, \omega, \cdot)\}_{t\geqslant 0, \omega\in\Omega}$ defined on a metric dynamical system $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ with state space l^2 is said to be asymptotically null in $\mathcal{D}(l^2)$, if for a.e. $\omega \in \Omega$, $D(\omega) \in \mathcal{D}(l^2)$, and any $\varepsilon > 0$, there exists $T(\varepsilon, \omega, D(\omega)) > 0$ and $M(\varepsilon, \omega, D(\omega)) \in \mathbb{N}$ such that

$$\sum_{|i|>M(\varepsilon,\omega,D(\omega))} \big|(\varphi\big(t, \theta_{-t}\omega, u(\theta_{-t}\omega)\big)_i\big|^2 \leqslant \varepsilon, \quad \forall t \geqslant T\big(\varepsilon, \omega, D(\omega)\big),\ \forall u(\omega) \in D(\omega).$$