

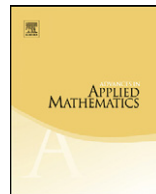


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On palindromic factorization of words

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ABSTRACT

Given a finite word u , we define its *palindromic length* $|u|_{\text{pal}}$ to be the least number n such that $u = v_1 v_2 \dots v_n$ with each v_i a palindrome. We address the following open question: let P be a positive integer and w an infinite word such that $|u|_{\text{pal}} \leq P$ for every factor u of w . Must w be ultimately periodic? We give a partial answer to this question by proving that for each positive integer k , the word w must contain a k -power, i.e., a factor of the form u^k . In particular, w cannot be a fixed point of a primitive morphism. We also prove more: for each pair of positive integers k and l , the word w must contain a position covered by at least l distinct k -powers. In particular, w cannot be a Sierpinski-like word.

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1. Introduction

Let A be a finite non-empty set, and let A^+ denote the set of all finite non-empty words in A . A word $u = u_1 u_2 \dots u_n \in A^+$ is called a *palindrome* if $u_i = u_{n-i+1}$ for each $i = 1, \dots, n-1$. In particular each $a \in A$ is a palindrome. We also regard the empty word as a palindrome.

Palindrome factors of finite or infinite words have been studied from different points of view. In particular, Droubay, Justin and Pirillo [4] proved that a word of length n can contain at most $n+1$ distinct palindromes, which gave rise to the theory of *rich* words (see [5]). The number of palindromes of a given length occurring in an infinite word is called its *palindrome complexity* and is bounded by a function of its usual subword complexity [1]. However, in this paper we study palindromes in an infinite word from the point of view of decompositions.

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For each word $u \in A^+$ we define its *palindromic length*, denoted by $|u|_{\text{pal}}$, to be the least number P such that $u = v_1 v_2 \dots v_P$ with each v_i a palindrome. As each letter is a palindrome, we have $|u|_{\text{pal}} \leq |u|$, where $|u|$ denotes the length of u . For example, $|01001010010|_{\text{pal}} = 1$ while $|010011|_{\text{pal}} = 3$. Note that 010011 may be expressed as a product of 3 palindromes in two different ways: $(0)(1001)(1)$ and $(010)(0)(11)$. In [10], O. Ravsky obtains an intriguing formula for the supremum of the palindromic lengths of all binary words of length n . The question considered in this paper is

Question 1. Do there exist an infinite non-ultimately periodic word w and a positive integer P such that $|u|_{\text{pal}} \leq P$ for each factor u of w ?

We conjecture that such aperiodic words do not exist, but at the moment we can prove it only partially. Namely, in this paper we prove that if such a word exists, then it is not k -power-free for any k and moreover, for all $k > 1, l \geq 0$ it does not satisfy the (k, l) -condition defined in Section 4. A discussion what exactly the condition means and which class of words should be studied now to give a complete answer to the question is given in Section 5.

A preliminary version of this paper has been reported at Journées Montoises 2012.

2. The case of k -power-free words

Let k be a positive integer. A word $v \in A^+$ is called a k -power if $v = u^k$ for some word $u \in A^+$. An infinite word $w = w_1 w_2 \dots \in A^{\mathbb{N}}$ is said to be k -power-free if no factor u of w is a k -power. For instance, the Thue–Morse word 0110100110010110... fixed by the morphism $0 \mapsto 01, 1 \mapsto 10$ is 3-power-free (see for example [7]).

Theorem 1. Let k be a positive integer and $w = w_1 w_2 \dots \in A^{\mathbb{N}}$. If w is k -power-free, then for each positive integer P there exists a prefix u of w with $|u|_{\text{pal}} > P$.

Recall that a word $u_1 \dots u_n$ is called t -periodic if $u_i = u_{i+t}$ for all i such that $1 \leq i \leq n - t$. The proof of Theorem 1 will make use of the following lemmas.

Lemma 2. Let u be a palindrome. Then for every palindromic proper prefix v of u , we have that u is $(|u| - |v|)$ -periodic.

Proof. If u and v are palindromes with v a proper prefix of u , then v is also a suffix of u and hence u is $(|u| - |v|)$ -periodic. \square

In what follows, the notation $w[i..j]$ can mean the factor $w_i w_{i+1} \dots w_j$ of a word $w = w_1 \dots w_n \dots$ as well as its precise occurrence starting at the position numbered i ; we always specify it when necessary.

Lemma 3. Suppose the infinite word w is k -power-free. If $w[i_1..i_2]$ and $w[i_1..i_3]$ are palindromes with $i_3 > i_2$, then

$$\frac{|w[i_1..i_3]|}{|w[i_1..i_2]|} > 1 + \frac{1}{k-1}.$$

Proof. By Lemma 2, the word $w[i_1..i_3]$ is $(i_3 - i_2)$ -periodic; at the same time, it cannot contain a k -power, so, $|w[i_1..i_3]| < k(i_3 - i_2)$. Thus,

$$\frac{|w[i_1..i_3]|}{|w[i_1..i_2]|} = \frac{|w[i_1..i_3]|}{|w[i_1..i_3]| - (i_3 - i_2)} > \frac{|w[i_1..i_3]|}{(1 - \frac{1}{k})(|w[i_1..i_3]|)} = 1 + \frac{1}{k-1}. \quad \square$$

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