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A polynomial algorithm of edge-neighbor-scattering number of trees

Yong Liu[a], Zongtian Wei[a,*], Jiarong Shi[a], Anchan Mai[b]

[a] School of Science, Xi'an University of Architecture and Technology, Xi'an, Shaanxi 710055, PR China
[b] Border Defense College of the PLA of China, Science-Cultural Institute, Xi'an, Shaanxi 710108, PR China

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ABSTRACT

The edge-neighbor-scattering number (ENS) is an alternative invulnerability measure of networks such as the vertices represent spies or virus carriers. Let $G = (V, E)$ be a graph and e be any edge in G. The open edge-neighborhood of e is $N(e) = \{f \in E(G) | f \neq e, e$ and f are adjacent$\}$, and the closed edge-neighborhood of e is $N[e] = N(e) \cup \{e\}$. An edge e in G is said to be subverted when $N[e]$ is deleted from G. An edge set $X \subseteq E(G)$ is called an edge subversion strategy of G if each of the edges in X has been subverted from G. The survival subgraph is denoted by G/X. An edge subversion strategy X is called an edge-cut-strategy of G if the survival subgraph G/X is disconnected, or is a single vertex, or is ϕ. The ENS of a graph G is defined as $ENS(G) = \max_{X \subseteq E(G)} \{\omega(G/X) - |X|\}$, where X is any edge-cut-strategy of G, $\omega(G/X)$ is the number of the components of G/X. It is proved that the problem of computing the ENS of a graph is NP-complete. In this paper, we give a polynomial algorithm of ENS of trees.

1. Introduction

Gunther and Hartnell [1,2] introduced the idea of modeling a spy network by a graph whose vertices represent the stations and whose edges represent links of communication. If a station is destroyed, the adjacent stations will be betrayed so that the betrayed stations become useless to network as a whole. Therefore, instead of considering the stability of a communication network in standard sense, some new graphical parameters such as vertex-neighbor-integrity [3] and edge-neighbor-integrity [4] were introduced to measure the stability of communication networks in the "neighbor" sense.

Recently, we study the spread of computer viruses in the Internet and biological viruses in the population and notice that networks in these background have a common property as that of the spy network. In order to measure the invulnerability of these networks (different from communication networks), we introduced edge-neighbor-scattering number (ENS) in [5] and vertex-neighbor-scattering number (VNS) in [6]. It is shown that the ENS and its vertex analogue (VNS) are alternative invulnerability measures of networks we mentioned above.

It is well known that a network can be described by a connected graph. The common of the above parameters is that, when removing some vertices (or edges) from a graph, all of their adjacent vertices (or edges) are removed. Therefore, we call ENS and VNS neighbor invulnerability parameters.

* Corresponding author. Tel.: +86 15353635611; fax: +86 2982205670.
E-mail address: wzt6481@163.com (Z. Wei).

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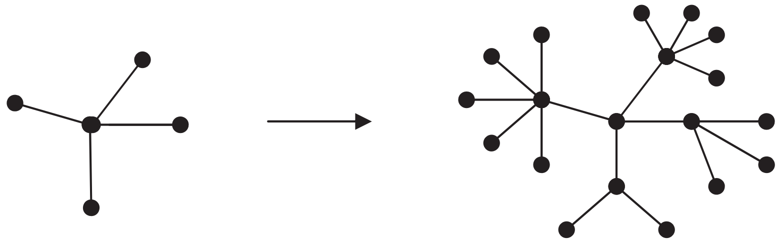

Fig. 1. A star and the corresponding star-tree.

Let $G = (V, E)$ be a graph and e be an edge of G. The *open edge-neighborhood* of e is $N(e) = \{f \in E(G) | f \neq e, e$ and f are adjacent$\}$, and the *closed edge-neighborhood* of e is $N[e] = N(e) \cup \{e\}$. An edge e of G is said to be *subverted* when $N[e]$ is deleted from G. In other words, if $e = [u, v]$, then $G - N[e] = G - \{u, v\}$. An edge set $X \subseteq E(G)$ is called an *edge-subversion-strategy* of G if each of the edges in X has been subverted from G. The survival subgraph is denoted by G/X. An edge subversion strategy X is called an *edge-cut-strategy* of G if the survival subgraph G/X is disconnected, or is a single vertex, or is ϕ.

Let G be a connected graph. The *edge-neighbor-scattering number* of G is defined as $ENS(G) = \max_{X \subseteq E(G)} \{\omega(G/X) - |X|\}$, where X is any edge-cut-strategy of G, and $\omega(G/X)$ is the number of the components of G/X. We call $X^*(\subseteq E(G))$ an *ENS-set* of G if $ENS(G) = \omega(G/X^*) - |X^*|$.

We have proved that the problem of computing the ENS of a graph is NP-complete [7]. In this paper, we give a polynomial algorithm of ENS of trees –a class of special and important graphs. Throughout this paper, we use Bondy and Murty [8] for terminologies and notations not defined here.

2. Preliminaries

Before proceeding, we define several concepts which will be used in the follows.

Let T be a tree. A vertex $v \in V(T)$ is called an *out-twig-vertex* if v is adjacent to at least two 1-degree vertices and adjacent to at most one non-1-degree vertex. The set of whole out-twig-vertices of T is denoted by $C(T)$. For $v \in C(T)$, the set of all 1-degree vertices adjacent to v is denoted by $N_T^+(v)$. A *leaf* of a tree T is a vertex $v \in V(T)$ with degree 1 and the degree of its adjacent vertex is 2. Let G be a graph and $e \in E(G)$. If the degree of one end-vertex of e is 1, we then call e a *pendant edge*.

A *star-tree* is a tree by replace each edge of a star with stars. A star and the corresponding star-tree are shown as follows. (See Fig. 1).

Let G be a graph and $e \in E(G)$. The *subdivision* of e is such an operation that delete e from G and connect its two end-vertices by an internal disjoint path with distance at least 2. If at least one edge in graph G is subdivided, then we call the resulting graph a *subdivision graph* of G.

Definition 1. Let $F = \bigcup_{i=1}^{k} T_i$ be a forest, where T_i is the branch tree of F. Then the edge-neighbor-scattering number of F is defined to be $ENS(F) = \bigcup_{i=1}^{k} ENS(T_i)$.

Lemma 1. *Let $S_{1,n}^*$ be a subdivision graph of star $S_{1,n}$. If delete all the leaves of $S_{1,n}^*$ layer by layer from outside to inside and denote the final star by $S_{1,n}'$, then $ENS(S_{1,n}^*) = ENS(S_{1,n}') + 1 = n - 2$.*

Proof. It is easy to know that for any $X \subseteq E(S_{1,n}^*)$, $\omega(S_{1,n}^*/X) \leq n - 2 + |X|$. Therefore, we have

$$ENS(S_{1,n}^*) \leq n - 2 + |X| - |X| = n - 2.$$

On the other hand, there exists an edge e in $S_{1,n}^*$ such that the degree of its one end-vertex is $n - 1$ and the degree of another end-vertex is 2. Since $\omega(S_{1,n}^*/\{e\}) = n - 1$, we have

$$ENS(S_{1,n}^*) \geq n - 1 - 1 = n - 2.$$

The second equation is trivial. The proof is completed. □

Lemma 2. *Let T be a tree which is not isomorphic to a path or a star. If e is a pendant edge of T, then there exists an ENS-set of T, X, such that $e \notin X$.*

Proof. Let $e = uv$ be a pendant edge of T and $d(u) = 1$. Suppose that X is an ENS-set of T, we distinguish the following cases.

Case 1. $d(v) = 2$. Assume another neighbor vertex of v is w. Since T is not isomorphic to a path or a star, $d(w) \geq 2$. Let $X' = X - uv$. If $w \notin V(T[X])$, then $\omega(T/X') = \omega(T/X)$; if $w \in V(T[X])$, then $\omega(T/X') = \omega(T/X) + 1$. Therefore,

$$\omega(T/X') - |X'| \geq \omega(T/X) - |X'| = \omega(T/X) - |X| + 1 = ENS(T) + 1,$$

a contradiction.