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Statistical tracking behavior of affine projection algorithm for unity step size

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ABSTRACT

Since unity step size could guarantee the fastest convergence and more detailed analysis for the affine projection (AP) algorithm, a statistical tracking behavior of AP algorithm is discussed in this paper. Deterministic recursive equations are derived for the mean weight error and mean-square error. All the possible correlations between the adaptive filtering coefficients and the past measurement noise are considered as well.

1. Introduction

Over the past decades, many computationally efficient, rapidly converging adaptive filtering algorithms have been numerously proposed across a myriad of engineering realms. Among all these algorithms, the affine projection (AP) algorithm, discovered from the geometric viewpoint of affine subspace projections, attracts particular attention, both theoretical and experimental [1]. Based on the direction vector, a new definition for the AP algorithm was presented when the unity step size was incorporated [2]. To achieve both the fastest convergence rate and the lowest steady-state error, the optimal step size AP algorithm was also discussed [3]. By setting the weight error to be zero in the direction of the adaptive weight update, the optimal step size was obtained for the pseudo-AP (PAP) algorithm [4]. By considering the measurement noise influence on the steady-state error, the first moment estimation of the measurement noise was used to modify the variable step size update, and then a modified variable step size affine projection sign algorithm was proposed in [5]. Along this framework, there still exist a great number of excellent progresses, such as normalized least mean square algorithm with orthogonal factors [6], AP with direction error algorithm [7, 8], AP algorithm dynamically selecting input vectors [9], to name yet a few.

Except for the accumulated achievements of updating of algorithms itself, analysis of its statistical properties also becomes a hot issue recently. Here we can mention some examples for capturing a clearer knowledge. In Ref. [10], a statistical analytical model for predicting the stochastic behavior of the AP algorithm has been provided for autoregressive (AR) inputs. To extend applicability of input processes that is suitable AR as well as autoregressive-moving average, a new statistical analysis framework was added to the AP algorithm with unity step size [11]. If the step size of PAP algorithm was less that one, deterministic recursive equations were derived for the mean weight error and for the mean-square error (MSE) [12,13]. Moreover, a quantitative analysis of the AP algorithm was presented [14], which analyzed the mean weight error and the MSE based on an independent and identically distributed input signal. In [15,16], the correlations between the weight error and the past measurement noise was presented to analyze the AP algorithm.

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In spite of great effort, the research of tracking behavior in AP gets limited attention. In addition, since unity step size yields the fastest convergence and permits a more detailed analysis for AP algorithm, statistical tracking behavior of the AP algorithm is discussed in the present work. The deterministic recursive equations are derived for the mean weight error and MSE. We show that, because of the property of the direction vector, the correlations between the weight error and the past measurement noise discussed in [15] become zero.

2. AP algorithm

In the system identification mode of the adaptive filter, the input process is converted into input vector $\mathbf{x}_n$, and it is defined as

$$\mathbf{x}_n = [x_n \quad x_{n-1} \quad \dots \quad x_{n-N+1}]^T \tag{1}$$

Based on the most recent m past input vectors, the input matrix $\mathbf{X}_{n-1,m}$ is defined as

$$\mathbf{X}_{n-1,m} = [\mathbf{x}_{n-1} \quad \mathbf{x}_{n-2} \quad \dots \quad \mathbf{x}_{n-m}] \tag{2}$$

The adaptive filter implements the AP algorithm with unity step size [1,2], as follows:

$$\hat{d}_n = \mathbf{w}_n^H \mathbf{x}_n \tag{3a}$$

$$e_n = d_n - \hat{d}_n \tag{3b}$$

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \frac{e_n^*}{\boldsymbol{\varphi}_n^H \boldsymbol{\varphi}_n} \boldsymbol{\varphi}_n \tag{3c}$$

where the direction vector $\boldsymbol{\varphi}_n$ is evaluated according to

$$\boldsymbol{\varphi}_n = \mathbf{x}_n - \mathbf{X}_{n-1,m} \hat{\boldsymbol{\alpha}}_n \tag{3d}$$

And the vector $\hat{\boldsymbol{\alpha}}_n = [\ \hat{a}_{n,1} \quad \hat{a}_{n,2} \quad \dots \quad \hat{a}_{n,m}\ ]^T$ is found based on the least-squares formulation

$$\hat{\boldsymbol{\alpha}}_n = \left(\mathbf{X}_{n-1,m}^H \mathbf{X}_{n-1,m}\right)^{-1} \mathbf{X}_{n-1,m}^H \mathbf{x}_n \tag{3e}$$

Substituting (3e) into (3d), we obtain

$$\boldsymbol{\varphi}_n = \left[\mathbf{I} - \mathbf{X}_{n-1,m}\left(\mathbf{X}_{n-1,m}^H \mathbf{X}_{n-1,m}\right)^{-1} \mathbf{X}_{n-1,m}^H\right] \mathbf{x}_n \tag{4}$$

Pre-multiplying (4) by $\mathbf{X}_{n-1,m}^H$, results in

$$\mathbf{X}_{n-1,m}^H \boldsymbol{\varphi}_n = \mathbf{0} \tag{5}$$

2.1. Statistical properties of the direction vector

The tracking analysis of the AP algorithm is done based on the following assumptions:

A1. The direction vector $\boldsymbol{\varphi}_n$ is independent of the weight vector $\mathbf{w}_n$ and the other direction vector $\boldsymbol{\varphi}_l, l \neq n$, and identically distributed with covariance matrix

$$\mathbf{R}_\varphi = E\left[\boldsymbol{\varphi}_n \boldsymbol{\varphi}_n^H\right] = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^H \tag{6}$$

where $\boldsymbol{\Lambda} = diag[\ \lambda_1 \quad \lambda_2 \quad \dots \quad \lambda_N\ ]$ and $\mathbf{V} = [\ \boldsymbol{\upsilon}_1 \quad \boldsymbol{\upsilon}_2 \quad \dots \quad \boldsymbol{\upsilon}_N\ ]$. The latter eigenvectors are orthonormal each other. This assumption has been justified in [10]. Also, it can be obtained from (3c).

A2. The direction vector $\boldsymbol{\varphi}_n$ is the product of three independent random variables that are independent and identically distributed. That is

$$\boldsymbol{\varphi}_n = s_n r_n \mathbf{v}_n \tag{7}$$

and

$$\begin{cases} P(s_n = \pm 1) = \frac{1}{2} \\ r_n \sim \|\boldsymbol{\varphi}_n\| \\ P(\mathbf{v}_n = \boldsymbol{\upsilon}_i) = \frac{\lambda_i}{\mathbf{tr}(\mathbf{R}_\varphi)};\ 1 \leq i \leq N \end{cases} \tag{8}$$

where $r_n \sim \|\boldsymbol{\varphi}_n\|$ means that r_n has the same distribution as the norm of the true direction vectors, and $\mathbf{tr}(\bullet)$ is adopted to denote the trace of a matrix. Similar assumption has been made for the input vector $\mathbf{x}_n$ in [14,15,17].

A3. The desire signal d_n is given by the model

$$d_n = \mathbf{w}_n^{oH} \mathbf{x}_n + \varepsilon_n \tag{9}$$