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Approximation of parametric curves by Moving Least Squares method

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ABSTRACT

In this work we propose a method to approximate a parametric curve in $\mathbb{R}^d$. Some distinct points in $\mathbb{R}^d$ are given, we assume that these points belong to a parametric curve and our aim is to approximate these data by Moving Least Squares method. We mention several applications of the proposed method to emphasize the importance of the work, also Root Mean Squares errors and Hausdorff distances between the exact curve and its approximation are presented to demonstrate the efficiency and reliability of the method.

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1. Introduction

Moving Least Squares (MLS) method is using in several parts of modern sciences. This method as an approximation method has been introduced by Shepard [34], in the lowest order case and generalized to higher degree by Lancaster and Salkauskas [21].

We need to fit a curve to an irregularly spaced set of points in many science and engineering problems. Curve fitting has been studied extensively in Approximation Theory and Geometric Modeling, and there are numerous books on the subject [2,9,11,12,21]. Existing techniques typically find a single curve segment that approximates or interpolates a function at some given points. Many techniques assume that the points are ordered and fit a curve to them by minimizing an error criterion [20,31].

Parametric curves are widely used in different fields such as Computer Aided Geometric Design (CAGD), Computer Graphics (CG), Computed Numerical Control (CNC) systems [14,28]. One basic problem in the study of parametric curves is to approximate the curve with lower degree curve segments. For a given digital curve, there exist methods to find such approximate curves efficiently [1,19,29,30]. Recently there have been presented so many works associated to parametric and non-parametric curves and their application. In these works we can see how to reconstruct or approximate and compute intersections of parametric curves and use the proposed methods in some fields of medical and engineering sciences [4,7,10,15,16,18,22,23,26,32,33,36,37,39].

Curves can be represented in parametric or implicit form in general but parameterizing a curve have availed some important advantages of being bounded in parameter range, independent of coordinate system over implicit representation. Furthermore, programming of parametric curves is easier and shorter in length. Also MLS method is a powerful method for approximating scattered data and one of the most attractive properties of this method is the local behavior.

A main category of parametric curves are Bézier curves that have prominent role and applications in mathematics and engineering sciences [6,17,27]. In Computer-Aided Design and Computer-Aided Manufacturing (CAD/CAM) and data

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compression the degree reduction problem of Bézier curves that means how approximate a given Bézier curve of a high degree by a new one of low degree, has many application [3,5,13]. Despite the goal and structure of our proposed MLS method is different but we have a comparison between MLS method and multi-degree reduction or degree reduction methods which proposed by Chen et al. in [5] and Bogacki et al. in [3].

The structure of this paper is as follows: in Section 2 we reintroduce the Moving Least Squares method in general form. In Section 3 we use MLS method to approximate parametric curves. In Section 4 the applications of proposed method in some practical situations are presented. There are some numerical examples in Section 5. The paper is ended with a brief conclusion in Section 6.

2. Moving Least Squares

Suppose that $\Omega \subset \mathbb{R}^d$ be a specific domain, and u be an unknown multivariate function on Ω which the values of u are known over a finite set of points $X = \{x_1, x_2, \dots, x_N\} \subseteq \Omega$. We want to approximate u at a certain point x in Ω. The MLS as an approximation method consists of three components: A basis function, a weight function associated with each node, and a set of coefficients that depend on node position. For a fixed point x we take

$$u(x) \simeq \hat{u}(x) = P^T(x)\boldsymbol{\alpha} = \sum_{j=1}^{m} \alpha_j p_j(x), \quad x \in \Omega, \tag{2.1}$$

where $P(x) = (p_1(x), p_2(x), \dots, p_m(x))^T$ is a m-dimensional basis of functions and the coefficient vector $\boldsymbol{\alpha}$ is a vector of parameters to be determined. $\boldsymbol{\alpha}$ is determined by solving a minimization problem which is defined as follows:

$$\min\{M(\boldsymbol{\alpha}) : \boldsymbol{\alpha} \in \mathbb{R}^m\}, \tag{2.2}$$

where M is a quadratic function with respect to m-tuple $\boldsymbol{\alpha}$,

$$M(\boldsymbol{\alpha}) = \sum_{i \in I_{x,\delta}} \left(u(x_i) - \sum_{j=1}^{m} \alpha_j p_j(x_i) \right)^2 w(x - x_i),$$

and

$$I_{x,\delta} = \{i \in \{1, 2, \dots, N\} : \|x - x_i\|_2 \le \delta\},$$

also $w : \mathbb{R}^d \to \mathbb{R}$ is a nonnegative weight function with support in the unit ball $B(0, 1)$ and is positive on the ball $B(0, 1/2)$. The parameter δ belongs to $\mathbb{R}^+$ and it is usually called *smoothing length* or *dilatation parameter* in the meshfree literatures.

Let Ω_x be a δ neighborhood of a fixed point $x \in \Omega$. The problem (2.2) can be written as the following matrix form

$$\min_{\boldsymbol{\alpha} \in \mathbb{R}^m} \left\{ M(\boldsymbol{\alpha}) = U_{\Omega_x} W_{\Omega_x} U_{\Omega_x} - 2\boldsymbol{\alpha}^T P_{\Omega_x} W_{\Omega_x} U_{\Omega_x} + \boldsymbol{\alpha}^T P_{\Omega_x} W_{\Omega_x} P_{\Omega_x}^T \boldsymbol{\alpha} \right\}, \tag{2.3}$$

where

$$\begin{cases} \boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_m)^T, \\ U_{\Omega_x} = (u(x_i) | i \in I_{x,\delta})^T \in \mathbb{R}^{c_x}, \\ P_{\Omega_x} = (p_j(x_i))_{i \in I_{x,\delta},\ j \in \{1,2,\dots,m\}} \in \mathbb{R}^{m \times c_x}, \\ W_{\Omega_x} = \operatorname{diag}(w(x - x_i) | i \in I_{x,\delta}) \in \mathbb{R}^{c_x \times c_x}, \end{cases} \tag{2.4}$$

in which c_x is the cardinal of $I_{x,\,\delta}$. Suppose that with suitable conditions, $\boldsymbol{\alpha}^*$ be the solution of (2.3), we can use the necessary condition $\nabla M(\boldsymbol{\alpha}) = 0$ to find $\boldsymbol{\alpha}^*$. Thus it implies that

$$\left[P_{\Omega_x} W_{\Omega_x} P_{\Omega_x}^T\right] \boldsymbol{\alpha} = P_{\Omega_x} W_{\Omega_x} U_{\Omega_x}. \tag{2.5}$$

The system (2.5) is a system of m linear equations with m unknowns. The cloud of neighbors in Ω_x should fulfill certain good neighborhood requirements to prevent that the matrix of coefficients in (2.5) be singular, it means that the number of points inside of Ω_x must be greater than the number of the basis functions m [8,24,25]. We substitute $\boldsymbol{\alpha}^*$ the solution of the system (2.5) in (2.1) and achieve an approximation to function u in the neighborhood of x [35].

3. Approximating a parametric function by MLS method

Let a set of distinct points $X = \{x_1, x_2, \dots, x_N\}$ in $\mathbb{R}^d$ is given. In this section we want to construct a locally parametric curve that approximately passes some certain x_j from X by MLS method.

Suppose that $C(t) = (f_1(t), f_2(t), \dots, f_d(t))$ be a parametric function such that $C(t_j) = x_j$ for $j = 1, 2, \dots, N$. Without lose of generality assume that for $j = 1, 2, \dots, N$, $t_j \in [0, 1]$ and they are distinct. So we have the following data

$$f_i(t_j) = \gamma_{i,j} = x_j^{(i)}, \begin{cases} i = 1, 2, \dots, d, \\ j = 1, 2, \dots, N, \end{cases}$$

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