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Effects of limited medical resource on a Filippov infectious disease model induced by selection pressure

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ABSTRACT

In reality, the outbreak of emerging infectious diseases including SARS, A/H1N1 and Ebola are accompanied by the common cold and flu. The selective treatment measure for mitigating and controlling the emerging infectious diseases should be implemented due to limited medical resources. However, how to determine the threshold infected cases and when to implement the selective treatment tactics are crucial for disease control. To address this, we derive a non-smooth Filippov system induced by selective treatment measure. The dynamic behaviors of two subsystems have been discussed completely, and the existence conditions for sliding segment, sliding mode dynamics and different types of equilibria such as regular equilibrium, pseudo-equilibrium, boundary equilibrium and tangent point have been provided. Further, numerical sliding bifurcation analyses show that the proposed Filippov system has rich sliding bifurcations. Especially, the most interesting results are those for the fixed parameter set as the bifurcation parameter varies, the sliding bifurcations occur sequentially: crossing → buckling → real/virtual equilibrium → buckling → crossing. The key factors which affect the selective treatment measures and the threshold value of infected cases for emerging infectious disease have been discussed in more detail.

1. Introduction

In the last few years, frequent outbreaks and quick spread of the emerging infectious disease become a worldwide public healthy problem. It endangers not only people's health but also the stability of the whole society. In 2003, SARS (Severe Acute Respiratory Syndrome) killed 774 people, infected more than 8000 globally and threatened to spread around the world [1,2]. The A/H1N1 (Hemagglutinin 1, Neuraminidase 1) influenza virus, which caused the 2009 pandemic, continues to circulate in some parts of the world, causing variable levels of disease and outbreaks [3]. By September 14, 2014, a total of 4507 probable and confirmed cases, including 2296 deaths from Ebola virus disease (EVD) had been reported from five countries in West Africa-Guinea, Liberia, Nigeria, Senegal, and Sierra Leone [4]. In the early stages of the emerging infectious disease, it is deficient to recognize the emergence of these infectious systematically and comprehensively, the measures of

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disinfection and isolation are failed to protect and control infected people, so the patients have little resistance to infection of any kind and emerging infectious spread through the camps like wildfire.

The emerging infectious disease including SARS, A/H1N1, Ebola, Dengue fever [5] etc. are often accompanied by other viral diseases such as common flu. Moreover, in the early stages of emerging infectious diseases (taking SARS as an example in the rest of paper) outbreak, there is only a very few number of individuals infected by SARS, and the medical resources are enough at this stage. Meanwhile, the early symptoms are very similar to flu and there is no effective way to identify the infected patients. Thus, patients infected with different viruses can be got treatment in different areas of the hospital at the same time.

With the growing numbers of SARS infected cases, those pose a grave threat to public health. Meanwhile, various kinds of control strategies have constraints based on the limited medical resources such as doctors, vaccines, drugs, hospital beds, isolation places, medical devices, and so on, especially in rural areas in many developing countries [6–10]. The medical resource limitation seriously restricts the prevention and treatment for SARS. At this moment, the department of health or state has to cite the urgency of fighting SARS, adopts green passage policy that speeds for isolation and treatment for SARS, so the doctors have to focus their attentions on the SARS infected cases only. For the patients with common flu, the doctors can only prescribe medicines and advise them to go home for home treatment, which can significantly relieve the pressure of limited medical resources on hospital or doctors.

In order to describe the effects of limited medical resource and selection strategy discussed above, the number of the patients infected by SARS in a compartment has been chosen as an index for hospital or doctors to use decisions. That is, if the number of the patients infected by SARS is below the threshold level which can be determined analytically (see main text for more details), there is no limited medical resource and selection pressure; above the threshold, due to the limited resource, and doctors treat SARS only. This type of control strategy is called as threshold control policy [11,12], which can be described by Filippov systems [13,14]. Recently, non-smooth Filippov infectious disease models have been investigated by many researchers [10,15–18].

In the present work, a non-smooth Filippov infectious disease model with threshold strategy induced by selective treatment measure is derived. The sliding mode dynamics and the existence of all types equilibria have been discussed. Numerical sliding bifurcation analyses show that the proposed Filippov system has rich sliding bifurcations. The key factors which affect the selective treatment measures and the threshold value of infected cases for emerging infectious disease have been discussed in more detail. Our main results show that reducing the threshold value to an appropriate level could contribute to the efficacy on prevention and treatment of emerging infectious disease, which indicates that the selection pressures can be beneficial to prevent the emerging infectious disease under medical resource limitation.

2. Models and threshold level

Let $S(t)$, $I_1(t)$, $I_2(t)$ and $R(t)$ denote the numbers of susceptible, the patients with SARS, common flu and recovered individuals at time t, respectively. For simplification, we assume that the people can only be infected either by SARS virus or by common flu virus. Further, based on the classical infectious disease model with limited capacity for treatment [9,10,19–25] we propose the following SI_1I_2R model as the basic model in this study

$$\begin{cases} \dot{S}(t) = A - \mu_S S - \beta_1 S I_1 - \beta_2 S I_2, \\ \dot{I}_1(t) = \beta_1 S I_1 - \mu_1 I_1 - \nu_1 I_1 - \dfrac{p_1 c_1 I_1}{1 + p_1 b_1 I_1 + p_2 b_2 I_2}, \\ \dot{I}_2(t) = \beta_2 S I_2 - \mu_2 I_2 - \nu_2 I_2 - \dfrac{p_2 c_2 I_2}{1 + p_1 b_1 I_1 + p_2 b_2 I_2}, \\ \dot{R}(t) = \nu_1 I_1 + \nu_2 I_2 + \dfrac{p_1 c_1 I_1 + p_2 c_2 I_2}{1 + p_1 b_1 I_1 + p_2 b_2 I_2} - \mu_R R. \end{cases} \tag{2.1}$$

The parameters of model (2.1) are summarized in Table 1. Obviously, two probabilities $p_1, p_2 \in [0, 1]$. Note that some very special cases of model (2.1) have been studied in our previous work [10], and the main purpose in this work is to investigate the generalized cases and reveal the rich dynamics and important biological implications concerning emerging infectious disease control.

It follows from model (2.1) that the total recovery rate

$$\frac{p_1 c_1 I_1 + p_2 c_2 I_2}{1 + p_1 b_1 I_1 + p_2 b_2 I_2} \tag{2.2}$$

is the major concern for the doctors once the emerging infectious disease outbreaks. Intuitively, how to choose the treatment proportions p_1 and p_2 for the patients infected by different virus such that the total recovery rate reaches its maximal value? To address this question, we discuss the selective strategies in the following.

Conditional upon resource limitation, we first assume that medical treatment service for SARS is more than the patients infected by flu, that is

$$\frac{c_1}{b_1} > \frac{c_2}{b_2}. \tag{2.3}$$