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# Numerical solution of steam flow in a nozzle using different non-equilibrium condensation models

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ABSTRACT

The paper presents three Eulerian models for the two-phase flow of a steam with a non-equilibrium condensation due to the rapid expansion. All models are based on the transport equations for the mass, momentum and total energy of the mixture. The models differ in the number of additional transport equations for the parameters of liquid phase. The models with two and four additional transport equations take into account homogeneous nucleation and growth of existing droplets. The last model with no additional transport equation is based on a "switch" from metastable to equilibrium state, i.e. a "switch" from zero to equilibrium wetness. Although this last model omits the droplet size, it can be particularly interesting for the simplified flow simulations in the first steps of steam turbine design (e.g. simulations of the circumferentially averaged flow in a meridional plane of several turbine stages). Presented numerical results of one- and two-dimensional flows in a convergent–divergent nozzle have been obtained using in-house codes based on the symmetrical operator splitting with a finite volume method used for the convection and a Runge–Kutta method used for time integration of source terms. The result discussion covers the comparison of three presented models in terms of Mach number, pressure and wetness prediction. It further covers the influence of grid density on the prediction of nucleation zone as well as a new thermodynamic closure alternative to the IAPWS-95 formulation.



## 1. Introduction

The first steps of steam turbine design rely on iterative solvers, which are generally based on the coupling between geometry, thermodynamics and loss models, see e.g. [1,13]. These solvers use finite volume formulation for the circumferentially averaged three-dimensional transport equations unlike to their predecessors, which were based on streamline curvature or stream function methods and which were not able to handle precisely transonic flow. The discretization of the solution domain (typically several turbine stages) often uses a rather coarse structured grid. The thermodynamic properties of steam are obviously approximated by the perfect gas model adjusted to the specific range of temperatures and pressures or using the equilibrium steam model according to the IAPWS-95 release. Many current numerical as well as experimental studies of the flow in the parts of steam turbines are still using single phase flow models or air as an alternative fluid, e.g. [2]. The present paper introduces a simple model of non-equilibrium condensation, which considers an expansion of steam starting either above saturation curve (super-heated region) or slightly below this curve (metastable, sub-cooled region) and which switch into wet equilibrium state

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**Nomenclature**

| | |
|---|---|
| $e$ | total energy (J m$^{-3}$) |
| $H$ | enthalpy (J kg$^{-1}$) |
| $J$ | nucleation rate (m$^{-3}$ s$^{-1}$) |
| $k_B = 1.3806 \times 10^{-23}$ | Boltzmann constant (J K$^{-1}$) |
| $Kn$ | Knudsen number (–) |
| $L$ | latent heat of evaporation/condensation (J kg$^{-1}$) |
| $m_1 = 2.99046 \times 10^{-26}$ | mass of one molecule of water (kg) |
| $N$ | total number of droplets (–) |
| $p$ | pressure (Pa) |
| $r$ | droplet radius (m) |
| $R$ | gas constant (J kg$^{-1}$K$^{-1}$) |
| $s$ | entropy (J kg$^{-1}$K$^{-1}$) |
| $S = p/p_s$ | super-saturation ratio (–) |
| $t$ | time (s) |
| $T$ | temperature (K) |
| $u, v$ | velocity components (m s$^{-1}$) |
| $U$ | internal energy (J kg$^{-1}$) |
| $x, y$ | spatial coordinates (m) |

**Greek letters**

| | |
|---|---|
| $\chi$ | wetness (mass fraction of liquid phase) (–) |
| $\lambda$ | thermal conductivity (W m$^{-1}$ K$^{-1}$) |
| $\mu$ | dynamic viscosity (Pa s) |
| $\rho$ | density (kg m$^{-3}$) |
| $\sigma_\infty$ | surface tension (N m$^{-1}$) |

**Subscripts**

| | |
|---|---|
| $0$ | total (reservoir) |
| $c$ | critical |
| $\ell$ | liquid |
| $s$ | saturated |
| $v$ | vapor |

(two-phase flow), when a given sub-cooling is reached. Such simple model does not require any additional transport equation, therefore it can be relatively easily implemented into existing single-phase flow code and it can approximate the effect of latent heat release due to the non-equilibrium condensation. This model is within the paper denoted as the 0-equation model. The results achieved by the 0-equation model are compared with the results of other non-equilibrium models with either two additional transport equations for the wetness and the total number of droplets, see e.g. [4,15] or with four additional transport equations for the moments, see e.g. [6]. The 0-equation model is not able to simulate the width of the nucleation zone, since the latent heat is released in "a single point". It may result in pressure overshoot, which can be also observed for the other models in the case of refined grid. The effect of grid refinement is presented for the case of one-dimensional flow in a nozzle. The part of the paper is devoted to the equation of state for the steam. Authors, Aungier and Dykas et al. [11,16], look for a compromise between perfect gas model and IAPWS-95 [12] (or IAPWS IF-97) model. The former model is simple and fast, but it requires e.g. additional empirical correction for nucleation rate and it yields bigger errors for higher pressures. The later describes well the properties of steam, but it is very slow from the point of view of CPU time. The results obtained using recently introduced "EOS for CFD purposes" model [14] are compared with the results obtained using classical IAPWS-95 model.

## 2. Flow models

All considered flow models have the form of the set of hyperbolic partial differential equations of the first order,

$$\frac{\partial}{\partial t}(W_k) + \frac{\partial}{\partial x}(F_k(W)) + \frac{\partial}{\partial y}(G_k(W)) = P_k(W), \quad k = 1, \ldots, k_{\max}, \tag{1}$$

where $W_k$ is the $k$th component of the vector of unknowns $W$ and $F_k$, $G_k$ and $P_k$ are the $k$th components of fluxes and a production term. The system (1) consists of transport equations for the parameters of the whole mixture (vapor with homogeneously dispersed fine droplets) and additional parameters of the droplet spectra. The first four equations of the system (1) are common for all models and describe the conservation of mass, $x$- and $y$-momentum and total energy of the mixture. The first four components