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An improved fitness evaluation mechanism with noise in prisoner's dilemma game

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ABSTRACT

The prisoner's dilemma game is a simple model to understand the evolution of cooperation in complex systems. In this work, we introduce the element of noise to the payoffs of the spatial prisoner's dilemma game and study its effects on the evolution of cooperations. A tunable parameter, termed as the noise strength (α), is introduced into the model to mimic the effects of noise in the individual fitness calculation. Our modified model remarkably promotes the behavior of cooperation, which may help us to better understand the emergence of cooperation in natural, social and economical systems.

1. Introduction

The emergence and maintenance of cooperation not only appear on human societies, but also appear on bacteria and animal. Cooperative behaviors promote the development of human society [1–3]. Evolutionary games have become a powerful tool to address cooperative behaviors [4,5]. A variety of game models are proposed to investigate the evolution and emergence of cooperation, such as prisoner's dilemma game (PDG) [6–12], the snowdrift game (SDG) [13,14] and public goods game (PGG) [15,16]. Among which mentioned above, the PDG game has been used as a basic model for social dilemmas and become a paradigmatic model to explore the evolution of cooperation.

In the original PDG, two involved players are simultaneously asked to make a choice between cooperating and defecting. If they choose to cooperate they will receive the highest collective payoff, which will be shared evenly among them and both of them will receive a reward R. Mutual defection, on the other hand, yields the lowest collective payoff and both of them will receive a punishment P. If a player defects while the opponent chooses to cooperate, the former receives a temptation T, and the latter receives a sucker's payoff S. The ranking of these four payoffs is $T > R > P > S$. This implies that players are more prone to defect if they both wish to maximize their own payoffs, regardless of the opponent's decision. The resulting is a social dilemma inducing the widespread defection, which is inconsistent with the fact that cooperative and altruistic behaviors are widely observable in real life. Following the notation suggested in Refs. [17,18], we utilize the rescaled payoff matrix: the temptation to defect $T = b$ (the highest payoff received by a defector playing with a cooperator), reward for mutual cooperation $R = 1$, and both the punishment for mutual defection P and the sucker's payoff S (the lowest payoff required by a cooperator encountering a defector) is equal to 0, $P = S = 0$. The condition $1 \le b \le 2$ ensures the proper payoff ranking.

Nowadays, the evolutionary game theory mainly follows two research lines and has made great progress. On the one hand, the network reciprocity is a well-known dynamical rule that fosters the prevalence of cooperation [19]. It says that if the game

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players are arranged on a network, where the individuals occupy the network nodes and the links determine who interacts with whom, the cooperators can shape compact clusters to prevent the invasion of defectors [17,20,21]. Thus the number of cooperators will preserve at a high level. Nowak and May [22] proposed the first model of the networking PDG, where the players were located on the square lattices. At each round of the dynamical process, players first gathered their own payoffs via the neighboring interactions based on the regulation of PDG. Then each player had a chance to adopt the strategy of his neighbors if they had higher payoffs. By simply taking the interaction structure into consideration, the cooperative behavior can be prevalent. On the other hand, the central question in evolutionary game theory is to find mechanisms and conditions that result in cooperation among selfish individuals. Till now, multifarious mechanisms have been proposed, such as kin selection, retaliating behavior, reward and punishment [23–25], voluntary participation [26,27], spatially structured population [28–36], heterogeneity or diversity [37], the mobility of players [38–40], age structure [41,42], and so on.

Presently, we report a new mechanism for the promotion of cooperation based on square lattice network. In the original PDG, it is assumed that the players have perfect rationality. However, from the perspective of behavioral economics, human beings are far from being rational calculators, as they often make mistakes, and behave irrationally [43]. Therefore, at each round of the game, players may not have the ability to obtain their payoffs exactly, due to the presence of noise derived from factors such as the error of observation. Concerning the diversity or discrepancy of people [44–46], the degree of noise for different players may have some heterogeneity. With the above arguments, we extend the traditional PDG by introducing the element of noise into the definition of fitness of individuals. We assume that the noise among different players are independent. During the strategy updating stage at each round of the game, the presence of noise impacts the strategy switching for each player according to the imitation dynamics. Following this research direction, many researches on prisoner's dilemma game have proved to be fruitful. Gaussian and Lévy distributions of stochastic payoff have been reported in [47,48], slowly varying small-world topology and additive spatiotemporal random variations have been introduced to the payoffs of a spatial prisoner's dilemma game in [49,50], noise-induced cooperation promotion in the spatial prisoner's dilemma game has been presented in [51–53], heterogeneous coupling between interdependent lattices promotes the cooperation in the prisoner's dilemma game and coupling effect on the evolution of cooperation based on the traveler's dilemma game have been discussed in [54,55], the individual diversity and increasing neighborhood size on two interdependent lattices has been investigated in [56], and a weak prisoner's dilemma where each player's participation is probabilistic rather than certain has been introduced in [57]. These authors all focus on the two involved players' payoffs and consider the noise effects on the PDG. Different from these models, here we only introduce noise to one involved player and discuss its impacts on the evolutionary game of cooperative behavior. To make a comparison with the original PDG, we still arrange the players on regular lattice networks following Ref. [22]. We extensively perform the computer simulations to elucidate the impact of different levels of noise. In the following part of this paper, we first specify our modified model of PDG; subsequently, we present the main results; and at last, we will summarize the conclusions.

2. Model

Let us consider an evolutionary prisoner's dilemma game, where the players occupy the nodes of a regular $L \times L$ square lattice network with the periodic boundary condition. Each player (each node), e.g. x, is initially assigned to be either a cooperator ($s_x = C$) or a defector($s_x = D$) with the equal probability which is described as

$$s_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad or \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

According to the theory mentioned in the above, the element of payoff matrix can be rescaled: $R = 1, P = S = 0$, and $T = b(1 < b < 2)$, and can be expressed by a matrix

$$\varphi = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}. \tag{2}$$

In the original PDG governed by the imitation dynamics, a player updates his strategy according to the following rules: at each round, the player x performs the PDG with each of his four nearest neighbors. In this way he gathers the resulting gains as his total payoff:

$$p_x = \sum_{y \in \Omega_x} s_x^T \varphi s_y \tag{3}$$

where Ω_x represents the four neighboring participants of x. Then he randomly selects one neighboring player y, and measures the difference of their payoff at this round, round. With this method, he decides whether to change his own strategy with a probability based the on the Fermi function

$$W(s_x \to s_y) = \frac{1}{1 + \exp[(p_x - p_y)/K]}, \tag{4}$$

where K is the intensity of selection [58], and p_x, p_y are the payoff of players x, y, respectively.

Every player in our model will be affected by external noises just as people are inevitably affected by environment. In reality, players may not measure their payoffs exactly and moreover it is more difficulty to acquire the actual payoffs of its adjacent