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A convex total generalized variation regularized model for multiplicative noise and blur removal

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ABSTRACT

Multiplicative noise and blur corruptions usually happen in coherent imaging systems, such as the synthetic aperture radar. Total variation regularized multiplicative noise and blur removal models have been widely studied in the literature, which can preserve sharp edges of the recovered images. However, the images recovered from the total variation based models usually suffer from staircase effects. To overcome this deficiency, we propose a total generalized variation regularized convex optimization model. The resulting objective function involves the total generalized variation regularization term, the MAP based data fitting term and a quadratic penalty term which is based on the statistical property of the noise. Indeed, the MAP estimated data fitting term in the multiplicative noise and blur removal model is nonconvex. Under a mild condition, the quadratic penalty term makes the objective function convex. A primal-dual algorithm is developed to solve the minimization problem. Numerical experiments show that the proposed method outperforms some state-of-the-art methods.

1. Introduction

Image denoising and deblurring are very fundamental but still challenging tasks in many imaging systems. The acquired images are usually corrupted by blur and noise simultaneously which severely degrade the quality of images. The additive Gaussian noise is the most common noise appearing in the incoherent imaging systems. Extensive researches have been done for the additive Gaussian noise and blur removal [1–4]. However, the proposed methods for additive Gaussian noise and blur removal cannot be directly applied to the multiplicative noise and blur removal problems. The multiplicative noise is image dependent and usually appears in the coherent image-acquisition systems, such as synthetic aperture radar (SAR), ultrasound imaging systems, laser imaging systems, and so on [5–7]. Suppose Ω is a connected open, bounded subset of $\mathbb{R}^2$ with Lipschitz boundary. Let u be the original image which is defined on Ω, i.e., $u : \Omega \to \mathbb{R}$, the process of multiplicative noise and blur degradation can be formulated as:

$$f = (Hu)\eta,$$

where $f : \Omega \to \mathbb{R}$ is the observed image. Usually we assume that $u > 0$. In this paper, we assume that η follows a gamma distribution with mean 1, which usually appears in SAR. In the multiplicative noise removal problem, H is the identity operator. In the simultaneous multiplicative noise and blur removal problem, H is a spatial-invariant blurring matrix. Note that the specific

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structure of the blurring matrix H is determined by the boundary conditions. For instance, if the periodic boundary condition is used, H is a matrix of block circulant with circulant blocks (BCCB). When the reflective boundary condition is applied, H is a matrix with block Toeplitz-plus-Hankel with Toeplitz-plus-Hankel blocks (BTHTHB) structure [8].

The multiplicative noise and blur removal is a typical ill-posed inverse problem. So far, many approaches have been proposed to solve this problem. Specially, total variational regularized approaches have attracted great interest. The first total variation based model for the multiplicative noise and blur removal was proposed by Rudin et al. [9]. They proposed the following constrained minimization model:

$$\begin{aligned} &\min_{u\in BV(\Omega)} \int_\Omega |Du| dx \\ &\text{subject to } \int_\Omega \frac{f}{Hu} dx = 1, \\ &\int_\Omega \left(\frac{f}{Hu} - 1\right)^2 dx = \sigma^2, \end{aligned} \tag{1}$$

where σ^2 stands for the variance of η and $\int_\Omega |Du| dx$ is the total variation of u [9]. Aubert and Aujol [10] proposed another total variation regularized model (AA) for multiplicative noise removal:

$$\min_{u\in BV(\Omega)} \int_\Omega \left(\log u + \frac{f}{u}\right) dx + \lambda \int_\Omega |Du| dx, \tag{2}$$

where the data fitting term is derived from maximum a posteriori (MAP) estimation according to multiplicative gamma noise degradation model and $\lambda > 0$ is a regularization parameter which is used as the trade-off between the regularization term and the data fitting term. The AA model was originally proposed for the multiplicative noise removal case but it can be extended to deal with blur removal. However, the minimization problem (2) is nonconvex, it may stick at some local minima. To overcome this deficiency, Huang et al. [11] suggested keeping the data fitting term of the AA model but enforcing total variation regularization in the logarithm transformation of the image domain i.e., $\int_\Omega |D(\log u)| dx$ was used as the regularization term in the AA model. Moreover, they obtained a convex optimization model by replacing $\log u$ with an auxiliary variable ω:

$$\min_{\omega\in BV(\Omega)} \int_\Omega (\omega + fe^{-\omega}) dx + \lambda \int_\Omega |D\omega| dx. \tag{3}$$

The recovered image is further acquired via exponential transformation of the solution ω^*, i.e., $u^* = e^{\omega^*}$. Their numerical experiments demonstrated this proposed method provides better solution than the AA model. However, since the logarithm transformation is nonlinear, the dark areas in the image are expanded while the corresponding bright parts are compressed [12].

Steidl and Teubner proposed a different convex variational model involving the Kullback–Leibler (KL) divergence data fidelity term and the TV or nonlocal mean regularization term [13]. The model is expressed as:

$$\min_u \int_\Omega (u - f \log u) dx + \lambda \Phi(u), \tag{4}$$

where $\Phi(u)$ is TV or nonlocal mean regularization term. Recently, Dong and Zeng proposed a new convex variational model for multiplicative noise and blur removal, which consists of a MAP estimated data fitting term, a quadratic term, and the TV regularization [14]. The details of this model will be reviewed in Section 3 as our model is inspired by their method.

Although total variation regularized models admit many desirable properties of the solutions, especially the preservation of the significantly sharp edges, they also give rise to undesired staircase effects in the smooth transition regions of image [12]. The reason is that TV based models assume the solutions are piecewise constant. To alleviate the staircase artifacts and preserve the sharp edges, total generalized variation (TGV) proposed by Bredies et al. [15] has attracted much interest in the image science. TGV assumes an image consists of piecewise polynomial functions which indeed remove the staircase effect. TGV has already been applied in the additive Gaussian noise, blur removal and compressive imaging [16,17]. Recently, TGV has been incorporated with the KL-divergence term to remove the speckle noise [12]. However, they did not consider the deblurring case.

In this paper, we propose a convex total generalized variation based model for blur and noise removal which is inspired by the work in [14]. The proposed model includes three terms: the TGV regularizer, the data fitting term estimated by MAP and a quadratic penalty term based on the statistical property of gamma noise. It is worth noting that the difference between the approach in [14] and the proposed method is that the former uses TV as regularizer while the latter utilizes the TGV regularization. The TGV is utilized to alleviate the undesired staircase artifacts while preserving sharp edges of the images. Furthermore, the existence of the solution of our proposed model is discussed in Section 3.

The rest of this paper is organized as follows. In Section 2, we present a brief introduction of TGV. In Section 3, we give a brief review of the convex relaxation model proposed in [14] and propose a new convex TGV regularized model. In Section 4, we present the numerical scheme based on primal-dual algorithm to solve the proposed model. In Section 5, the numerical experiments are presented to demonstrate the effectiveness of the proposed method. Finally, conclusions are given in Section 6.