Applied Mathematics and Computation 276 (2016) 182–199

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Generalized Taylor polynomials for axisymmetric plates and shells

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ARTICLE INFO

Keywords:
Taylor polynomials
Collocation method
Plates
Shells
Axisymmetric problems

ABSTRACT

This work proposes the use of a mesh-free technique, derived from the generalized Taylor polynomials, for the analysis of axisymmetric plates and shells. The primary solution variable(s) is/are assumed to take the form of a truncated Taylor series around a point *c*, and the unknown coefficients of the expansion are determined using the governing differential equation(s) and boundary conditions. The method is free of shape-parameter calibration needed in some other famous mesh-free techniques such as the RBF, and is quite easy to formulate and program. Successful application of the method to several benchmark problems of axisymmetric plate and shell structures proves its robustness. The results have been verified using the existing rigorous analytical solutions that are in most cases not suited to practical engineering calculations.

1. Introduction

Taylor series is widely used for representing functions satisfying certain conditions. Depending on the nature of the function, the number of terms needed to achieve acceptable results varies. It has proved useful in several applications. Numerical solution of differential equations by collocation using Taylor series to approximate the primary solution variable(s) is one of such. This approach, also referred to as the Taylor collocation method (TCM), is purely meshless and unlike other famous meshless methods such as the RBF [1–9], it doesn't require the use of any calibration parameter that affects accuracy.

Some previous works conducted using the Taylor collocation technique include that of Sezer [10], where approximate solution of the second-order linear differential equation with specified associated conditions is reported. Its application on generalized Hermite, Laguerre, Legendre and Chebyshev equations proves successful. Collocation using the Taylor polynomials has proved useful in solving integro-differential equations by several researchers. Karamete and Sezer [11] used it to transform linear integro-differential equations to a system of linear algebraic equations in which the unknowns are the Taylor coefficients. The method is appraised based on its performance on certain linear differential, integral, and integro-differential equations. Solution of general high-order linear Fredholm–Volterra integro-differential equations have been reported in [12–16] using the Taylor collocation method by transforming the integral equation to a matrix form via the collocation points. The analysis presented in [16] suggested that the method has a very rapid convergence rate. A closely similar work has been reported in [17] for linear integro-fractional differential equations of Volterra-type of order $n\alpha$ for $0 < \alpha \leq 1$. Higher-order linear complex differential equations in the elliptic domains have been solved by Sezer et al. [18], using a computer program written in Maple9, based on the Taylor collocation method and its validity tested using some illustrative examples. Solution of high-order nonhomogeneous difference equations

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http://dx.doi.org/10.1016/j.amc.2015.12.003

has been obtained by Gülsu et al. [19] and by Gökmen and Sezer [20]. The obtained solutions, in terms of Taylor polynomials about any point, are for the equations having variable coefficients in both the aforementioned two references. Bagley–Torvik equation is solved by Çenesiz et al. [21] using the Taylor collocation method and its numerous advantages over other numerical methods for solving fractional differential equations have been highlighted. Solution, based on the same method, for high-order linear pantograph equations with linear functional argument has been presented in [22]. The properties of the method have been shown using illustrative examples consisting of initial conditions. Similar work is reported in [23] for pantograph functional differential equations with proportional delays of the first and higher orders. Both the initial and boundary-value problems can be conveniently handled by the method. Numerical solution of non-linear Schrodinger equation is obtained in [24] by using Taylor polynomials and cubic B-spline basis collocation method for time discretization and spatial discretization, respectively. The method was evaluated using some test problems. Dağ et al. [25] presented Taylor–Galerkin and Taylor collocation methods for the numerical solutions of Burgers' equation using B-splines. The Taylor series expansion is used in time discretization of the equation. Accuracy of the methods was assessed based on L_2 and L_∞ error norms. Delay integro-differential equations have been solved in [26,27] by collocation using Taylor polynomials and convergence of the method verified. Taştekin et al. [28] used the collocation method based on Taylor polynomials to solve a class of the first order nonlinear differential equations with mixed conditions. The high accuracy of the method, as well as its relative ease compared to some popular methods have been demonstrated. Approximate solution of delayed Lotka–Volterra predator-prey model has been obtained in [29]. A system of nonlinear equations involving the unknown Taylor coefficients is generated and solved.

Despite its appealing simplicity and remarkable accuracy, the collocation method in terms of Taylor polynomials has never, to the knowledge of the author, been applied to solve problems involving plates and shells. Exact solutions of the system of equations governing the behavior of this class of structural elements are, in most cases, rigorous and sometimes prohibitive for practical design purposes. For that reason, the common practice is to adopt approximate analytical or numerical simulation schemes in obtaining solutions. However, due to their limitations, the approximate analytical methods fail to perform in some cases. Solutions based on numerical methods are, therefore, more reliable. Hence, this paper proposes the use of a mesh-free technique, derived from the generalized Taylor polynomials, for the analysis of axisymmetric plates and shells. By satisfying the governing equation(s) and boundary conditions at a number of distributed nodes within the domain and at the boundary, respectively, we generate a matrix of the system of algebraic equations whose solution gives the values of unknown coefficients of the expansion.

2. Formulation

A general mth-order linear differential equation with variable coefficients takes the form given in Eq. (1) subject to some boundary conditions given by Eq. (2).

$$\sum_{k=0}^{m} P_k(x)y^{(k)}(x) = f(x), \quad a \le x \le b \tag{1}$$

$$\sum_{j=0}^{m-1} \left(\alpha_{ij}y^{(j)}(a) + \beta_{ij}y^{(j)}(b)\right) = \lambda_i, \quad i = 0,\ 1, \dots, m-1 \tag{2}$$

where $P_k(x), \quad k = 0,\ 1, \dots, m$ and $f(x)$ are functions of the space variable x, and α_{ij}, β_{ij} *and* λ_i are appropriate constants.

Eq. (3) is a candidate solution which is assumed to take the form of a Taylor series expansion around c where, as a priori, the function y is assumed to have nth derivative in the interval of expansion. The infinite series can be truncated at N terms that are sufficient for convergence. This is written in Eq. (4).

$$y(x) = \sum_{n=0}^{\infty} \frac{y^{(n)}(c)}{n!}(x-c)^n, \quad a \le x,\ c \le b, \tag{3}$$

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n, \quad a \le x,\ c \le b, \quad N \ge m \tag{4}$$

In order to find the unknown Taylor coefficients, a set of collocation points at some defined intervals within the problem domain is used thus.

$$a = x_0 < x_1 < \cdots x_{N-1} < x_N = b$$

For a uniform interval, we have

$$x_i = a + i\frac{b-a}{N}, \quad i = 0,\ 1,\ 2, \dots, N \tag{5}$$

Eq. (4) can be written in a matrix form thus

$$y(x) = XM_0A \tag{6}$$