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Analysis, adaptive control and circuit simulation of a novel nonlinear finance system

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A R T I C L E I N F O

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A B S T R A C T

In the last three decades a growing interest in developing nonlinear dynamical systems for economic models, displaying chaotic behavior has been developed. To this direction, a novel 3-D nonlinear finance chaotic system consisting of two nonlinearities is presented. The dynamical analysis of the proposed system confirms its complex dynamic behavior, which is studied by using well-known simulation tools of nonlinear theory, such as the bifurcation diagram, Lyapunov exponents and phase portraits. Also, some interesting phenomena related with nonlinear theory are observed, such as route to chaos through a period doubling sequence, antimonotonicity and crisis phenomena. In addition, an interesting scheme of adaptive control of finance system's behavior is presented. Furthermore, the novel nonlinear finance system is emulated by an electronic circuit and its dynamical behavior is studied by using the electronic simulation package Cadence OrCAD in order to confirm the feasibility of the theoretical model.

1. Introduction

In the last decades researchers from almost all fields of natural sciences have studied phenomena that involved nonlinear systems exhibiting chaotic behavior [1–4]. This is due to the fact that nonlinear systems demonstrate rich dynamics and have sensitivity on initial conditions. Chaotic phenomena have also been observed in economics by the middle 80's. From then till nowadays and especially in the last few years a new scientific field named "Econophysics" aims to study the complex dynamics of real economic systems. Towards this direction, researchers are trying to explain main features of economic theory such as structural changes, irregular (erratic) micro- and macro-economic fluctuations.

In order to study an economic model, as a first step, economists take into account only endogenous variables and so the economic model's behavior is simplified. Such an economic model, which lacks external excitation, is analogous to an autonomous electronic circuit. Then economists enrich the economic model with exogenous variables related with political events and physical disasters. For this reason, the complexity of these models makes accurate economic forecasting very difficult.

In many fields of economics, such as funding, stocks and social economics, the diversity and complexity are part of the internal structure of economic models that interact with external forces, economic or social, which have nonlinear

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factors. For this reason, nonlinear dynamics and especially chaotic systems have been introduced to the study of economic models. Examples of nonlinear systems used in economic models, are the well known van der Pol model [5,6] and others [7–14].

Even though, the French Mathematician Henri Poincaré studied for the first time chaotic systems at the end of the 19th century [15], chaos theory has begun to evolve in the second half of the 20th century [16,17] after observing different physical systems. Studying these systems revealed that although their evolution rules and initial conditions were well-known, their future progress seemed to be arbitrary and unpredictable. That was a great revolution in modern physics, terminating with Laplace's ideas of casual determinism [18].

Since then, chaos has been observed in many systems, such as weather and climate [19], population growth in ecology [20] and in laboratory as well in a number of systems, such as electrical circuits [21], lasers [22], chemical reactions [23], fluid dynamics [24], mechanical systems, and magneto-mechanical devices [25]. So, chaos can explain various phenomena not only in nature, but also in laboratory and many chaotic dynamical systems can be used in different scientific fields.

Chaos theory studies dynamical systems that evolve in time, presenting three very important features [26]:

- it must be topologically mixing,
- its periodic orbits must be dense and
- it must be very sensitive on initial conditions.

The first term, the topological mixing, means that the chaotic trajectory at the phase space will move over time so that each designated area of this trajectory will eventually cover part of any particular region. The second feature of chaotic systems, the fact that its periodic orbits have to be dense, means that the trajectory of a dynamical system is dense, if it comes arbitrarily close to any point in the domain. Furthermore, the third and probably the most important feature of chaotic systems, is the sensitivity on initial conditions. This means that a small variation on a system's initial conditions will produce a totally different chaotic trajectory.

Motivated by the complex dynamical behavior of chaotic systems, a novel nonlinear finance system is investigated, by using well-known tools of nonlinear theory, such as the bifurcation diagram, Lyapunov exponent and phase portrait. Various phenomena concerning chaos theory, such as the route to chaos through the mechanism of period doubling and crisis phenomena are observed. Also, an interesting scheme of adaptive control of system's behavior is presented. Furthermore, the proposed nonlinear finance system is emulated by an electronic circuit and its dynamical behavior is studied by using the electronic simulation package Cadence OrCAD.

This paper is organized as follows. In Section 2, the nonlinear finance system and its fundamental dynamics are presented. The dynamical behavior of the system is studied by numerical simulations in Section 3. The results for the global chaos control of the novel finance chaotic system with unknown parameters using adaptive control method is derived in Section 4, while in Section 5 the circuit which emulates the proposed system as well as the simulation results of system's behavior, by using the Spice are presented. Finally, Section 6 includes the conclusions of this work.

2. The novel nonlinear finance model

In 2001, a third-order dynamical model, describing a nonlinear finance system, was reported [10]. The model describes time variations of three state variables namely the interest rate x, the investment demand y, and the price index z. This nonlinear finance chaotic system is described by the following set of differential equations:

$$\begin{cases} \frac{dx}{dt} = z + (y - \alpha)x \\ \frac{dy}{dt} = 1 - by - x^2 \\ \frac{dz}{dt} = -x - cz \end{cases} \tag{1}$$

Parameters α, b and c stand for: the saving amount, the cost per-investment ratio and the elasticity of demand of commercial markets, respectively. All three parameters possess a positive value ($\alpha \geq 0$, $b \geq 0$, $c \geq 0$).

Also, the aforementioned finance system has been studied by other researchers in the same or in other mathematical forms [11,12,27], by extracting interesting results about its dynamical behavior.

In this work, the x^2 term, in the second equation of the system (1), has been replaced with $|x|$, which is more accurate from the economic point of view (see the second equation of system (2)). From the economic theory is known, that the key variable, which plays a crucial role in the investment process, is the interest rate. The changing rate of the investment demand y is in proportion by inversion with the interest rate x, as it has been reported in literature. So, by keeping the nonlinear nature of this term, the x^2 of system (1) has been replaced with $|x|$. Furthermore, the existence of $|x|$ in a finance system of this kind could explain better the real economic world, in which central banks in many countries try to keep the interest rates in small but positive values. From this point the proposed system is more close to a real finance system, because the $|x|$ ensures the aforementioned condition and for this reason, in this work the system