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Uncertain portfolio selection with background risk

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ABSTRACT

In real life, investors face background risk which may affect their portfolio selection decision. In addition, there are situations where background asset return and the security returns have to be given by experts' evaluations because of occurrence of unexpected incidents in economic and social environment or lack of historical data. This paper discusses an uncertain portfolio selection problem in which background risk is considered and the returns of the securities and the background assets are given by experts' evaluations instead of historical data. Using uncertainty theory, we propose a new uncertain portfolio selection model with background risk. To enable the users to solve the problem with currently available programing solvers, the crisp form of the model is provided. In addition, we discuss the optimal solution of the model when the returns of the securities and the background asset return obey normal uncertainty distributions, and compare the optimal portfolio with background risk with that without background risk. It is concluded that when everything else is same, the expected optimal portfolio return with background risk is smaller than that without background risk. Finally, a numerical example is given as an illustration.

1. Introduction

Portfolio selection is concerned with optimal allocation of one's capital to different assets in order to maximize the return of the portfolio for a given risk. Markowitz [31] first proposed the mean-variance model, which is the foundation of modern portfolio theory. In the mean-variance portfolio theory, the expected return is regarded as the investment return and variance the risk. However, the computational work of mean-variance model method is tremendous. In view of this, single factor model [38] and multi-factor model [37] were proposed. In addition, scholars proposed other alternative risk measures and based on them studied the balance methods of investment return and investment risk, e.g., mean-semivariance model [32], expected absolute deviation method [24], Value-at-Risk models [21,35], Conditional Value-at-Risk models [1,39], mean-semivariance-CVaR model [34], mean-risk curve model [13], etc.

These researches help investors well allocate their capital to a variety of securities from different perspectives. However, their assumptions are all that investors face only portfolio risk when making portfolio selection decisions. Yet, in reality, investors also face other sources of risk like those arising from variations in labor income, investments in real estate, and unexpected expenses related to health issues that cannot be hedged through portfolio diversification in the financial markets [4,8]. These sources of risk are usually referred to as background risk [2]. In the paper we refer to the assets that are exposed to background risk as background assets and securities in financial markets as financial assets. Background assets are typically illiquid or nontradable, and background risk is practically hard to be controlled by adjusting holdings of background assets in the short run. Many

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studies have shown that the presence of background risk can affect investments because it is the total risk rather than the sole portfolio risk that is of investors' major concern. For example, Heaton and Lucas [10,11] found that labor and entrepreneurial incomes affected portfolio allocations. Viceira [41] further pointed out that the optimal allocation to stocks was much larger for employed investors than for retired investors when selecting portfolio with nontradable labor income. Study of Rosen and Wu [36] showed that investors with bad health were more willing to put most of their wealth in the low risk asset rather than risky assets. Tsanakas [40] revealed that the presence of background risk made risk measurement sensitive to the scale and aggregation of risk. Research of Hara et al. [9] showed that background risk increased investors' cautiousness. To help investors select portfolio in face of background risk, scholars did many research works and a variety of models have been developed and discussed for optimal portfolio selection with background risk in different situations. For example, by using stochastic programing technique, Menoncin [33] provided solution methods for obtaining maximum expected exponential utility of investors' terminal real wealth. Baptista [2] offered conditions for investors to optimally delegate their wealth management to portfolio managers in face of background risk by minimizing tracking error variance for a given expected gain over the benchmark. Jiang et al. [22] analyzed and presented the properties of efficient portfolios and the location of the efficient frontier generated by financial assets with background risk, and Huang and Wang [12] further studied the portfolio frontier characteristics when risk-free asset is considered. In recent years, Baptista [3] introduced mental accounts into the problem and derived and compared the efficient frontier of the portfolios with mental accounts and background risk with the traditional efficient frontier of the portfolios. These studies all reveal that investors who consider background risk will become more risk averse and choose safer assets.

In the above works, returns of financial assets and background assets are all considered as random variables. However, from the essence of background risk we can see that there are situations where people lack suitable data about background asset returns and can only estimate their values. Then background asset returns contain much subjective imprecision rather than randomness. Regarding securities there are also situations where people have none or no sufficient historical data. For example, newly listed securities have few historical data and in unexpected events such as a surprising announcement of interest rate cut by central bank, etc., there are no suitable historical data either. Therefore, in these situations, people have to use their estimations expressed by belief degrees to describe the parameters of financial and background asset returns. According to the finding of Kahneman and Tversky [23], people give too much weight to unlikely invents, which implies that people's estimations usually contain a much wider range of values than the uncertain parameters may really take. If in this situation we still use probability theory to handle belief degrees, counterintuitive results may occur [29,43].

In order to deal with belief degrees, Liu [25] proposed an axiomatic uncertainty theory and further refined it [28]. By using uncertainty theory no counterintuitive results occur. In [30], it is shown that belief degrees satisfy the four axioms of uncertainty theory, which implies that it is suitable to use uncertainty theory to model people's estimations. Nowadays, uncertainty theory has been applied to many areas. For example, in the field of optimization, Liu [27] proposed a spectrum of uncertain programing models [28]. Especially, Huang [14] introduced uncertainty theory to portfolio selection in which security returns are given by experts' evaluations rather than historical data and produced an uncertain portfolio theory. After that, Huang proposed uncertain risk curve [15] and risk index [17], which contributed to risk control, and further investigated the extension of uncertain mean-variance model in depth [16]. Later, multi-period uncertain portfolio selection [18] and uncertain portfolio adjustment problem [19] based on risk index were studied. In addition, in the field of project selection, Zhang et al. [42] first applied uncertainty theory to a multinational project selection problem, later Zhang et al. [43] introduced a profit risk index and a cost overrun risk index and proposed an uncertain mean-risk index project selection model. Moreover, in the other fields, Gao [6] used uncertainty theory to solve a shortest path problem with uncertain lengths. Gao [7] employed uncertainty theory to deal with single facility location problems in which the vertex demand was uncertain, and Ding and Gao [5] handled uncertain multi-product newsboy problem, etc. In this paper, we will employ the uncertainty theory to study an uncertain portfolio selection problem with background risk in the situation where financial and background asset returns are all given by experts' estimations.

The paper proceeds as follows. In Section 2, we will introduce some necessary knowledge about uncertain variables for easy understanding of the paper. In Section 3, we will propose and discuss an uncertain portfolio selection model with background risk. In Section 4, we will provide and discuss a numerical example to illustrate significance of our approach. Finally, we will conclude the paper in Section 5.

2. Necessary knowledge about uncertain variable

Uncertainty theory is developed based on the following four axioms.

Definition 1. [25] Let L be a σ-algebra over a nonempty set Ω. We call every element $\Gamma \in L$ an event. A set function $M\{\Gamma\}$ is called an uncertain measure if it satisfies the following four axioms:

(i) (Normality) $M\{\Omega\} = 1$.
(ii) (Self-duality) $M\{\Gamma\} + M\{\Gamma^c\} = 1$.
(iii) (Countable subadditivity) For each countable sequence of events$\{\Gamma_i\}$, we have

$$M\left\{\bigcup_{i=1}^{\infty}\Gamma_i\right\} \leq \sum_{i=1}^{\infty} M\{\Gamma_i\}.$$

The triplet (Ω, L, M) is called an uncertainty space.