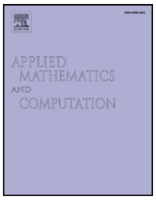
Applied Mathematics and Computation 276 (2016) 297–300

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc



Comments on a new class of nonlinear conjugate gradient coefficients with global convergence properties

Zhifeng Dai[*]

College of Mathematics and Computational Science, Changsha University of Science and Technology, Changsha 410114, China

ARTICLE INFO

MSC:
90C30
65K05

Keywords:
Unconstrained optimization
Conjugate gradient method
Global convergence

ABSTRACT

In Rivaie et al. (2012) [1], an efficient CG algorithm has been proposed for solving unconstrained optimization problems. However, due to a wrong inequality (3.3) used in Rivaie et al., the proof of Theorem 2 and the global convergence Theorem 3 are not correct. We present the necessary corrections, then the proposed method in Rivaie et al. still converges globally. Finally, we report some numerical comparisons.

1. Introduction

Due to low memory requirements and strong global convergence property, nonlinear conjugate gradient methods are efficient for solving the following unconstrained optimization problem,

$$\min f(x), \quad x \in R^n, \tag{1.1}$$

where f: $R^n \to R$ is a continuously differentiable function, especially if the dimension n is large.

The iterates of conjugate gradient methods for solving (1.1) are obtained by

$$x_{k+1} = x_k + \alpha_k d_k. \tag{1.2}$$

where α_k is a steplength. The steplength α_k is computed by carrying out some line search, and d_k is the search direction defined by

$$d_k = \begin{cases} -g_k, & \text{if } k = 1, \\ -g_k + \beta_k d_{k-1}, & \text{if } k \geq 2, \end{cases} \tag{1.3}$$

where β_k is a scalar, g_k is gradient of $f(x)$ at x_k. Varieties of this method differ in the way of selecting β_k.

In the recent paper [1], Rivaie et al. proposed a new class of nonlinear conjugate gradient coefficients which is called RMIL method. The parameter β_k in RMIL method is computed as follows

$$\beta_k^{\text{RMIL}} = \frac{g_k^T(g_k - g_{k-1})}{\|d_{k-1}\|^2} = \frac{g_k^T(g_k - g_{k-1})}{\|d_{k-1}\|^2}. \tag{1.4}$$

By using the following exact line search,

$$f(x_k + \alpha_k d_k) = \min_{\alpha \geq 0} f(x_k + \alpha d_k), \tag{1.5}$$

---

[*] Tel.: +86 731 85258285.
E-mail address: zhifengdai823@163.com

http://dx.doi.org/10.1016/j.amc.2015.11.085
0096-3003/© 2015 Elsevier Inc. All rights reserved.

the search direction d_k satisfies the sufficient descent condition:

$$g_k^T d_k = -\|g_k\|^2, \quad \forall k \geq 0. \tag{1.6}$$

Numerical comparisons show that this computational scheme outperforms some other conjugate gradient methods. However, due to a wrong inequality (3.3) used in Rivaie et al. [1], the proof of Theorem 2 and the global convergence Theorem 3 is not correct. In what follows, the necessary corrections will be presented.

2. Comments on the convergence of RMIL algorithm

Here, we firstly point out a wrong inequality used in Rivaie et al. [1], namely inequality (3.3) about β_k^{RMIL}, which plays a key role in global convergence analysis of Theorem 2 and Theorem 3.

In order to make the convergence proof easier, Rivaie et al. [1] first provided an upper bound for the coefficient β_k^{RMIL}. At (3.2) in [1], Rivaie et al. defined

$$\beta_{k+1}^{\mathrm{RMIL}} = \frac{g_{k+1}^T (g_{k+1} - g_k)}{\|d_k\|^2} = \frac{\|g_{k+1}\|^2 - g_{k+1}^T g_k}{\|d_k\|^2}, \tag{2.1}$$

and stated that

$$0 \leq \beta_{k+1}^{\mathrm{RMIL}} \leq \frac{\|g_{k+1}\|^2}{\|d_k\|^2}. \tag{2.2}$$

Since the sign of $g_{k+1}^T g_k$ in (2.1) cannot be identified as positive or negative, we can not obtain (2.2) ((3.3) in [1]). However, in the proof of Theorem 2 and Theorem 3 in [1], the inequality (2.2) ((3.3) in [1]) plays a critical role in global convergence analysis.

In order to accomplish the global convergence analysis of the RMIL method, we present a RMIL+ coefficient as follows

$$\beta_{k+1}^{\mathrm{RMIL+}} = \begin{cases} \dfrac{g_{k+1}^T (g_{k+1} - g_k)}{\|d_k\|^2}, & \text{if } \ 0 \leq g_{k+1}^T g_k \leq \|g_{k+1}\|^2, \\ 0, & \text{otherwise}. \end{cases} \tag{2.3}$$

It is obvious that $\beta_{k+1}^{\mathrm{RMIL+}}$ satisfies (2.2) ((3.3) in [1]).

Following the same proof as Theorem 2 and Theorem 3 in [1], we can get global convergence with exact line searches.

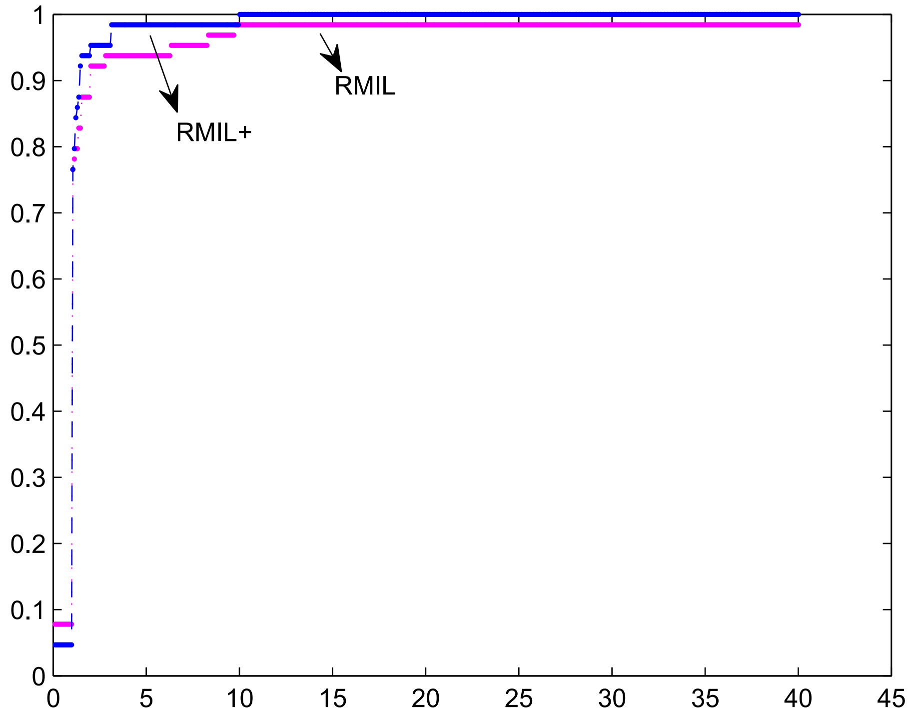


Fig. 1. Performance profiles with respect to CPU time.