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Multiplicity results for a p-Laplacian discrete problems of Kirchhoff type

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ABSTRACT

This paper is concerned with the existence and multiplicity solutions for a discrete boundary value problem of p-Kirchhoff type. Our technical approach is based on variational methods.

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1. Introduction

In this work, we study the existence and multiplicity of solutions for the following discrete problem with Neumann boundary conditions

$$\begin{cases} M(\|u\|^p)(-\Delta(\phi_p(\Delta u(k-1))) + q(k)\phi_p(u(k))) = \lambda f(k, u(k)), \quad k \in [1, T], \\ \Delta u(0) = \Delta u(T) = 0, \end{cases} \tag{1.1}$$

where, T is a fixed positive integer, $[1, T]$ denotes the discrete interval $\{1, 2, \dots, T\}$, $\Delta u(k-1) := u(k) - u(k-1)$ is the forward difference operator and λ is a positive parameter, ϕ_p will stand for the homeomorphism defined by $\phi_p(s) = |s|^{p-2}s,\ 1 < p < +\infty$. Also $q(k) \geq 0$ for all $k \in [1, T]$, $M(t)$ and $f : [1, T] \times \mathbb{R} \to \mathbb{R}$ are two continuous functions that satisfy some conditions which will be stated later on. Here, $\|.\|$ denote the norm

$$\|u\| := \left(\sum_{k=1}^{T+1} |\Delta u(k-1)|^p + \sum_{k=1}^{T} q(k)|u(k)|^p \right)^{1/p}$$

of the T-dimensional Banach space

$$X := \{u : [0, T+1] \to \mathbb{R} : \Delta u(0) = \Delta u(T+1) = 0\}.$$

As it is well known, problem (1.1) is related to the stationary problem of a model introduced by Kirchhoff [22]. More precisely, Kirchhoff introduced a model given by the following equation

$$\rho \frac{\partial^2 u}{\partial t^2} - \left(\frac{\rho_0}{h} + \frac{E}{2L} \int_0^L \left| \frac{\partial u}{\partial x} \right|^2 dx \right) \frac{\partial^2 u}{\partial x^2} = 0, \tag{1.2}$$

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where ρ, ρ_0, h, E, L are constants, which extends the classical D'Alembert's wave equation, by considering the effects of the changes in the length of the strings during the vibrations. A distinguishing feature of the Kirchhoff equation is that the equation contains a nonlocal coefficient $\frac{\rho_0}{h}+\frac{E}{2L}\int_0^L|\frac{\partial u}{\partial x}|^2dx$ which depends on the average $\frac{E}{2L}\int_0^L|\frac{\partial u}{\partial x}|^2dx$ of the kinetic energy $\frac{1}{2}\left|\frac{\partial u}{\partial x}\right|^2$ on $[0, L]$, and hence the equation is no longer a point wise identity. On the other hand, stationary counterpart of (1.2) is given as

$$\begin{cases}\left(a+b\int_\Omega|\nabla u|^2dx\right)\Delta u=f(x,u) & \text{in} \quad \Omega\\ u=0 & \text{on} \quad \partial\Omega,\end{cases} \tag{1.3}$$

which is has attracted much attention after the Lions paper [24], where a functional analysis frame work for the problem was proposed; see, e.g., [3,18,19,21] for some interesting results.

Discrete boundary value problems have been intensively studied in the last decade. The modeling of certain nonlinear problems from biological neural networks, economics, optimal control and other areas of study have led to the rapid development of the theory of difference equations; see the monograph of [1,2,17] and the references therein.

Equations involving the discrete p-Laplacian operator, subjected to classical or less classical boundary conditions, have been widely studied by many authors using various techniques. Recently, many results have been established by applying variational methods. In this direction we mention the papers [4–6,8,11,13–16,20,23,25,26]. However, to our knowledge, there is not a great number of papers which have dealt with discrete boundary value problems of Kirchhoff type. We refer the reader to [27,28] for an overview on this subject.

Our aim of this paper is to apply critical point theory to (1.1) and prove the existence of one, two, or three solutions. We assume throughout this paper that the Kirchhoff function $M(t)$ satisfies the following hypotheses:

(M_0) There is a positive constant m_0 such that

$$M(t)\geq m_0, \quad \text{for all } t\geq 0.$$

(M_1) There exists a positive constant ρ such that

$$\widehat{M}(t)\geq \rho M(t)t, \quad \text{for all } t\geq 0,$$

where $\widehat{M}(t)=\int_0^t M(s)ds$.

2. Preliminaries and basic notations

In this section, we state some basic properties, definitions and theorems to be used in this article. Let $(X, \|\cdot\|)$ be a finite dimensional Banach space. A functional I_λ is said to verify the Palais–Smale condition (in short (*P.S.*)) whenever one has that any sequence $\{u_n\}$ such that

(1) $\{I_\lambda(u_n)\}$ is bounded;
(2) $\{I'_\lambda(u_n)\}$ is convergent at 0 in X^*

admits a subsequence which is converging in X. Obviously, being X a finite dimensional Banach space, it is enough that any sequence $\{u_n\}$ satisfying (1) and (2) admits a bounded subsequence.

Now, we recall a result of local minimum obtained in [10], see also [9], which are based on variational methods on finite dimensional Banach spaces.

Theorem 2.1. *Let X be a finite dimensional Banach space and let $\Phi, \Psi : X \to \mathbb{R}$ two functions of class C^1 on X with Φ is coercive. In addition, suppose that there exist $r \in \mathbb{R}$ and $w \in X$, with $0 < \Phi(w) < r$, such that*

$$\frac{\sup_{\Phi^{-1}([0,r])}\Psi}{r}<\frac{\Psi(w)}{\Phi(w)}. \tag{2.1}$$

Then, for each

$$\lambda\in\Lambda_w:=\left[\frac{\Phi(w)}{\Psi(w)},\frac{r}{\sup_{\Phi^{-1}([0,r])}\Psi}\right],$$

the function $I_\lambda = \Phi - \lambda\Psi$ admits at least one local minimum $\overline{u} \in X$ such that $\overline{u} \neq 0$, $\Phi(\overline{u}) < r$, $I_\lambda(\overline{u}) \leq I_\lambda(u)$ for all $u \in \Phi^{-1}([0, r])$ and $I'_\lambda(\overline{u}) = 0$.

Theorem 2.2. *Let X be a finite dimensional Banach space and let $\Phi, \Psi : X \to \mathbb{R}$ two functions of class C^1 on X with Φ is coercive. Fix $r > 0$. Assume that for each*

$$\lambda\in\Lambda:=\left]0,\frac{r}{\sup_{\Phi^{-1}([0,r])}\Psi}\right[,$$

the function $I_\lambda = \Phi - \lambda\Psi$ satisfies the (PS)-condition and is unbounded from below. Then, for each $\lambda \in \Lambda$, the function I_λ admits at least two distinct critical points.

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