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A numerical approach with error estimation to solve general integro-differential–difference equations using Dickson polynomials

Ömür Kıvanç Kürkçü[a,*], Ersin Aslan[b], Mehmet Sezer[a]

[a] Department of Mathematics, Faculty of Science, Celal Bayar University, 45140 Manisa, Turkey
[b] Turgutlu Vocational Training School, Celal Bayar University, Manisa, Turkey

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ABSTRACT

In this paper, a matrix method based on the Dickson polynomials and collocation points is introduced for the numerical solution of linear integro-differential–difference equations with variable coefficients under the mixed conditions. In addition, in order to improve the numerical solution, an error analysis technique relating to residual functions is performed. Some linear and nonlinear numerical examples are given to illustrate the accuracy and applicability of the method. Eventually, the obtained results are discussed according to the parameter-α of Dickson polynomials and the residual error estimation.

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1. Introduction

In recent years, the nonlinear differential and integro differential–difference equations (IDDEs) have been studied in many application fields such as mechanics, engineering, astronomy, physics, chemistry, biology, economics, potential theory, electrostatics and electrodynamics [1–9,20,28]. Since the some nonlinear differential and IDDEs are generally difficult to solve analytically. For this reason, the numerical methods are required. On the other hand, the matrix methods based on Taylor, Chebyshev, Bernoulli, Bessel, Chelyshkov and Laguerre polynomials along with Adomian polynomials (Adomian decomposition method; ADM), He's variational iteration method (VIM) and He's homotopy perturbation method (HPM) have been given to find the approximate solutions of the differential–difference, delay differential, nonlinear differential and integro differential–difference equations [10–15,18,19,22–30,42–55]. Our purpose in this study is to develop and to apply the mentioned method to the m_1th-order IDDEs

$$\sum_{k=0}^{m_1}\sum_{j=0}^{J} P_{kj}(x)y^{(k)}(\lambda x+\mu) = g(x) + \int_a^b \sum_{r=0}^{m_2} K_r(x,t)y^{(r)}(t)dt + \int_a^{\psi(x)} \sum_{s=0}^{m_3} K_s(x,t)y^{(s)}(t)dt \tag{1a}$$

or briefly,

$$D_d(x) = g(x) + F(x) + V(x)$$

* Corresponding author. Tel.: +90 2362013202.
E-mail address: omurkivanc@outlook.com (Ö.K. Kürkçü).

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such that

$$D_d(x) = \sum_{k=0}^{m_1} \sum_{j=0}^{J} P_{kj}(x) y^{(k)}(\lambda x + \mu), \quad F(x) = \int_a^b \sum_{r=0}^{m_2} K_r(x,t) y^{(r)}(t) dt,$$
$$V(x) = \int_a^{\psi(x)} \sum_{s=0}^{m_3} K_s(x,t) y^{(s)}(t) dt \quad m_1 \geq m_2, m_3$$

and to the nonlinear differential equations in the form

$$P(x)y''(x) + P_0(x)y(x) + P_1(x)y'(x) + P_2(x)y^2(x) + P_3(x)y^3(x) = g(x) \tag{1b}$$

under the mixed conditions

$$\sum_{k=0}^{m_1-1} \left[a_{jk} y^{(k)}(a) + b_{jk} y^{(k)}(b) \right] = \delta_j, \quad j = 0, 1, 2, \ldots, m_1 - 1. \tag{2}$$

Here, the known functions $P_{kj}(x)$, $g(x)$, $K_r(x,t)$, $\psi(x)$, $K_s(x,t)$, $P(x)$ and $P_i(x)$ are defined on $-\infty < a \leq x, t \leq b < \infty$; a_{jk}, b_{jk} and δ_j are suitable constants. Our purpose is to obtain an approximate solution expressed in the truncated Dickson series form:

$$y(x) \cong y_N(x) = \sum_{n=0}^{N} y_n D_n(x, \alpha) \tag{3}$$

where y_n, $n = 0, 1, \ldots, N$ $(N \geq m_1)$ are the unknown Dickson coefficients. In order to find a solution in the form (3), we use the collocation points which are defined as follows:

$$x_i = a + \left(\frac{b-a}{N} \right) i, \quad i = 0, 1, 2, \ldots, N \tag{4}$$

where $a = x_0 < x_1 < \ldots < x_N = b$.

2. Basic properties of Dickson polynomials

The Dickson polynomials over finite fields were introduced by Dickson [16] and then were rediscovered by Brewer [17]. These polynomials are defined as polynomials over a commutative ring with identity and called as the first kind Dickson polynomials $\{D_0(x,\alpha), D_1(x,\alpha), \ldots, D_n(x,\alpha), \ldots\}$. These types of Dickson polynomials have several applications and interesting properties, the main one being a permutation property over finite fields.

The first kind Dickson polynomial $D_n(x,\alpha)$ of degree n with parameter $\alpha \in R$ is defined as [16,17,31–41]:

$$D_n(x,\alpha) = \sum_{p=0}^{\lfloor n/2 \rfloor} \frac{n}{n-p} \binom{n-p}{p} (-\alpha)^p x^{(n-2p)}, \quad n \geq 1, \quad -\infty < x < \infty.$$

Here, the initial polynomials are $D_0(x,\alpha) = 2$ and $D_1(x,\alpha) = x$. Also, R is a commutative ring with identity; $\lfloor n/2 \rfloor$ denotes the largest integer $\leq n/2$. For $\alpha = 0$, the Dickson polynomial $D_n(x,0)$ becomes the power polynomial x^n.

The polynomials $D_n(x,\alpha)$, $(n = 0, 1, 2, 3, \ldots)$ satisfy the following recurrence relation [37]:

$$D_n(x,\alpha) = xD_{n-1}(x,\alpha) - \alpha D_{n-2}(x,\alpha), \quad n \geq 2$$

where the two initial polynomials are $D_0(x,\alpha) = 2$ and $D_1(x,\alpha) = x$.

The polynomials $y = D_n(x,\alpha)$ satisfy the following second order differential equation:

$$(x^2 - 4\alpha)y'' + xy' - n^2 y = 0, \quad n = 0, 1, 2, 3, \ldots.$$

The $D_n(x,\alpha)$ has the generating function [37]

$$\sum_{n=0}^{\infty} D_n(x,\alpha) z^n = \frac{2 - xz}{1 - xz + \alpha z^2}.$$

Several combinatorial polynomial families and their dilates form subclasses of the Dickson polynomials; for instance, the Lucas (w-) polynomials $L_n(x)$ and the Pell–Lucas polynomials $Q_n(x/2)$ for $\alpha = -1$, the first kind Chebyshev polynomials $2T_n(x/2)$ for $\alpha = 1$ and the Fermat-Lucas polynomials $FL_n(x/3)$ for $\alpha = 2$ [37,38].

The Dickson polynomials $D_n(x,\alpha)$ play several important roles in the theory of finite fields, particularly in the study of permutation polynomials and Kloosterman sums. These applications arise via the well-known functional equation [37,39]

$$D_n\left(y + \frac{\alpha}{y}, \alpha\right) = y^n + \left(\frac{\alpha}{y}\right)^n$$

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