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L_2–L_∞ filtering for stochastic systems driven by Poisson processes and Wiener processes

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ABSTRACT

This paper investigates the L_2–L_∞ filtering problem for stochastic systems driven by Poisson processes and Wiener processes. Firstly, this paper presents an approach to transform the expectation of stochastic integral with respect to Poisson process into the expectation of Lebesgue integral by the martingale theory. Then, based on this, a filter is designed to guarantee that the filtering error system is mean-square asymptotically stable and its L_2–L_∞ performance satisfies a prescribed level. Finally, a simulation example is given to illustrate the effectiveness of the proposed filtering scheme.

1. Introduction

Since the state variables in control problems are not always available, state estimation is a very important problem. Kalman filtering scheme is one of the most effective ways of dealing with the state estimation problems. However, one drawback of Kalman filters is that the system model under consideration is required to be exactly known and the disturbances are restricted to be Gaussian noises with known statistics. When external noises are not precisely known, we can resort to the technique of L_2–L_∞ filtering or $\mathcal{H}_\infty$ filtering. The L_2–L_∞ performance was first discussed in [1]. The filtering problem based on such a performance is usually called energy-to-peak filtering and the main objective is to minimize the peak value of the estimation error for all possible bounded energy disturbances.

Stochastic phenomena are frequently encountered in many branches of science and engineering [2–9]. To model stochastic phenomena, researchers have employed Wiener processes widely in the past years [10–14]. However, Wiener process cannot describe jump stochastic phenomena effectively [15–17]. Nowadays, it has been recognized that Poisson process is a natural model for jump stochastic phenomena [18,19] and stochastic systems driven by Poisson processes can be found in many practical systems [18–23].

Considering that systems in the real world are often perturbed by continuous and jump stochastic phenomena simultaneously (e.g., multistage manufacturing system in [24]), researchers began to investigate stochastic systems driven by Poisson processes and Wiener processes. For instance, the optimal control problem for such stochastic systems was studied in [25]. Kolmanovsky and Maizenberg [26] investigated the optimal containment control problem for the following stochastic system

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perturbed by Poisson process and Wiener process:

$$dx(t) = f(x(t))dt + g(x(t))dw(t) + h(x(t))d\mathcal{N}(t), \tag{1}$$

where $w(t)$ is a Wiener process and $\mathcal{N}(t)$ is a Poisson process. However, according to [23] and [27], (1) should be improved by

$$dx(t) = f(x(t))dt + g(x(t))dw(t) + h(x(t-))d\mathcal{N}(t). \tag{2}$$

If investigating control or filtering problems for the system (2), we should utilize the Itô formula, which includes the following term:

$$\int_0^{\mathcal{T}} [V(t, x(t-) + h(x(t-))) - V(t, x(t-))]d\mathcal{N}(t). \tag{3}$$

(3) is a stochastic integral with respect to the semimartingale $\mathcal{N}(t)$ and $V(t, x(t))$ is a common Lyapunov function. It is very difficult to handle the expectation of this term directly. Moreover, the integrand of this integral is a function of $x(t-)$, which will increase the difficulty. Due to this reason, there is still no paper to discuss the filtering problem for stochastic systems driven by Poisson processes and Wiener processes. This motivates the present study.

In this paper, we are concerned with the L_2–L_∞ filtering problem for stochastic systems driven by Poisson processes and Wiener processes. Firstly, by utilizing the martingale theory, this paper transforms the expectation of stochastic integral with respect to Poisson process into the expectation of Lebesgue integral. Then, on the basis of this, we design a filter such that the filtering error system is mean-square asymptotically stable and its L_2–L_∞ performance satisfies a prescribed level. Finally, a simulation example is given to illustrate the effectiveness of the filtering scheme proposed in this paper.

Notation: In this paper, unless otherwise specified, we will employ the following notation. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ and $\mathbb{E}(\cdot)$ be the expectation operator with respect to the probability measure. If A is a vector or matrix, its transpose is denoted by A^T. If P is a square matrix, $P > 0 (P < 0)$ means that is a symmetric positive (negative) definite matrix of appropriate dimensions while $P \geq 0 (P \leq 0)$ is a symmetric positive (negative) semidefinite matrix. I stands for the identity matrix of appropriate dimensions. Let $|\cdot|$ denote the Euclidean norm of a vector and its induced norm of a matrix. Unless explicitly specified, matrices are assumed to have real entries and compatible dimensions. $L^2(\Omega)$ denotes the space of all random variables X with $\mathbb{E}|X|^2 < \infty$, it is a Banach space with norm $\|X\|_2 = (\mathbb{E}|X|^2)^{1/2}$. $\mathcal{L}_2[0, \infty)$ is the space of square integrable functions over $[0, \infty)$. The symbol '$*$' within a matrix represents the symmetric terms of the matrix, e.g.$\begin{pmatrix} X & Y \\ * & Z \end{pmatrix} = \begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix}$. If a function is right continuous with left limits, this function is called càdlàg function. If a function is left continuous with right limits, this function is called càglàd function. Moreover, a stochastic process is said to be càdlàg if it almost surely has sample paths which are right continuous with left limits. A stochastic process is said to be càglàd if it almost surely has sample paths which are left continuous with right limits.

2. Problem formulation and preliminaries

In this paper, we consider the following stochastic systems driven by Poisson processes and Wiener processes:

$$(\Sigma) : dx(t) = [Ax(t) + Bv(t)]dt + Cx(t)dw(t) + Dx(t-)d\mathcal{N}(t), \tag{4}$$

$$dy(t) = [Ex(t) + Fv(t)]dt, \tag{5}$$

$$z(t) = Lx(t), \tag{6}$$

$$x(0) = \xi, \tag{7}$$

where $x(t) \in \mathcal{R}^n$ is the state vector; $y(t) \in \mathcal{R}^q$ is the measurement; $v(t) \in \mathcal{R}^p$ is the disturbance input which belongs to $L_2[0, \infty)$; $z(t) \in \mathcal{R}^s$ is the signal to be estimated. A, B, C, D, E, F, L are known real constant matrices with appropriate dimensions. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ is a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$. $w(t)$ is a scalar standard Wiener process and $\mathcal{N}(t)$ is a one-dimension Poisson process with parameter $\lambda > 0$. $w(t)$ is independent of $\mathcal{N}(t)$.

Remark 1. As pointed out in [29, 30], stochastic differential equation (2) should be interpreted as meaning the corresponding stochastic integral equation:

$$x(t) = x(0) + \int_0^t f(x(s))ds + \int_0^t g(x(s))dw(s) + \int_0^t h(x(s-))d\mathcal{N}(s), \tag{8}$$

where $\int_0^t h(x(s-))d\mathcal{N}(s)$ is the stochastic integral with respect to the semimartingale $\mathcal{N}(s)$, whose definition can be found in [28,30].

For the system (Σ), we aim to design a filter of the following form:

$$(\Sigma_f) : d\hat{x}(t) = A_f\hat{x}(t)dt + B_f dy(t), \tag{9}$$