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Strang splitting method for Burgers–Huxley equation

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A B S T R A C T

We derive an analytical approach to the Strang splitting method for the Burgers–Huxley equation (BHE) $u_t + \alpha u u_x - \epsilon u_{xx} = \beta(1-u)(u-\gamma)u$. We proved that Srtang splitting method has a second order convergence in $H^s(\mathbb{R})$, where $H^s(\mathbb{R})$ is the Sobolev space and s is an arbitrary nonnegative integer. We numerically solve the BHE by Strang splitting method and compare the results with the reference solution.

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1. Introduction

Nonlinear partial differential equations are important in most fields of science. Burgers–Huxley equation has a great importance of being a nonlinear partial differential equation. It describes the interactions between reaction mechanisms, convection effects and diffusion transports [1]. This equation was firstly introduced by Bateman [2], then treated by Burgers [3] in a mathematical modeling of turbulence. These NPDEs are high importance in nonlinear physics. BHE is a perturbation problem by having the perturbation parameter $\epsilon \in (0, 1)$. To solve the BHE various numerical techniques were applied by the researchers. In [12], they have applied a quasilinearization process which is long and complicated. Also, Zhou and Cheng [13] have applied the operator splitting method to the BHE by solving two nonlinear subproblems. In [14], they propose a non-standard, finite difference scheme to approximate the solution of a generalized Burgers–Huxley equation. Also, in [15] a fourth order finite-difference scheme in a two-time level recurrence relation is proposed for the numerical solution of the generalized Burger–Huxley equation with stability analysis. The resulting nonlinear system is solved by predictor–corrector method. A higher order finite difference scheme [16] and B-spline collocation scheme [17] are implemented to find numerical solution of the generalized BHE.

In this work, we apply the Strang splitting method to the BHE and prove the convergence of the method theoretically in Sobolev spaces and numerically check the results. For the theoretical proof we follow the similar approach to [5] and [11]. Since the linear and nonlinear parts of the BHE have differences we apply the Strang splitting method and solve each subproblems easily. For the convergence analysis, we derive error bounds under given regularity results and properties of the Sobolev spaces. Our main aim is to prove the convergence rates in Sobolev spaces. We solve the BHE by dividing the problem into linear and nonlinear parts without doing any linearization.

In [4], the KdV equation is studied and they apply Lie–Trotter and Strang splitting in order to have error estimates for convergence. They actively make use of the fact that solutions of KdV equation remain bounded in a Sobolev space and this, together with a bootstrap argument guarantees the existence of a uniform choice of time step Δt that prevents the solution from blowing up. On the other hand, [5] studies equation with a Burgers type nonlinearity including the KdV equation. They

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make use of the fact that solutions of Burgers type equations remain bounded in a Sobolev space and perform an analysis which identifies error terms in the local error as quadrature errors which are estimated via Lie commutator bounds. In [6] and [7], similar analysis are studied for linear evolution equations and for nonlinear Schrödinger equations, respectively. The work is original with respect to the combination of the convergence analysis and computation.

This paper is organized as follows. In Section 2, we give a brief introduction to the Strang splitting method, introduce the model problem (BHE) then split the problem into linear and nonlinear parts. In Section 3, we present hypothesis about the local well-posedness and boundedness of the solution to the BHE then we deal with the regularity results for the BHE. Local and global error analysis in $H^s(\mathbb{R})$ are given in Section 4. Finally, we give the numerical results and prove the correct convergence rates for Strang splitting method.

2. Application the Strang splitting method to the BHE

The idea of operator splitting [8–10] is widely used for the approximation of partial differential equations. The basic idea is based on splitting a complex problem into simpler sub-problems, each of which is solved by an efficient method. One of the reasons for the popularity of operator splitting is the use of dedicated special numerical techniques for each of the equations.

We focus our attention on the case of linear and nonlinear operators such as,

$$u_t = Au(t) + B(u(t)), \ \text{with } t \in [0, T], \ u|_{t=t_0} = u_0 \tag{1}$$

We employ Lie–Trotter splitting method to the one-dimensional Burgers–Huxley equation,

$$u_t + \alpha u u_x - \epsilon u_{xx} = \beta(1-u)(u-\gamma)u, \tag{2}$$

with the initial condition

$$u|_{t=t_0} = u_0 \tag{3}$$

where $t > 0$, α, $\beta \geq 0$, $0 < \epsilon \leq 1$ and $0 < \gamma < 1$. When $\alpha = 0$ and $\epsilon = 1$, Eq. (2) reduces to Huxley equation and when $\beta = 0$, reduces to Burgers' equation.

With the help of the operator splitting, we break (2) into linear diffusion equation and nonlinear reaction equation. In this latter type of the operator splitting, the simpler equations are solved and then recoupled over the initial conditions in delicate ways to preserve a certain accuracy. The exact solution at the time t of (2) is given by $u(t) = \Phi^t_{A+B}(u_0)$ with given initial condition and the approximate split solution is denoted by u_n, at $t = n\Delta t \leq T$, as $\Delta t \to 0$, where $u_{n+1} = \Phi_A^{\Delta t/2}(\Phi_B^{\Delta t}(\Phi_A^{\Delta t/2}(u_n)))$, $n = 0, 1, 2, \ldots$.

In our case we split Eq. (2) into two subequations,

$$v_t = Av = \epsilon v_{xx} \tag{4}$$

and

$$w_t = B(w) = \beta(1-w)(w-\gamma)w - \alpha w w_x \tag{5}$$

acting on appropriate Sobolev spaces.

3. Error bounds and regularity results for Strang splitting

In the beginning of the analysis, we assume that the solutions to the BHE are locally well-posed and bounded. Thus, the following hypotheses are about the local well-posedness of the solutions to (2) and boundedness of the solution and initial condition in Sobolev spaces.

Hypothesis 3.0.1. For a fixed time T, there exists $M > 0$ such that for all u_0 in $H^k(\mathbb{R})$ with $\|u_0\|_{H_k} \leq M$, there exists a unique strong solution u in $C([0, T], H^k)$ of (2). In addition, for the initial data u_0 there exists a constant $K(M, T) < \infty$, such that

$$\|\tilde{u}(t) - u(t)\|_{H^k} \leq K(M, T)\|\tilde{u}_0 - u_0\|_{H^k}$$

for two arbitrary solutions u and $\tilde{u}$, corresponding to two different initial data $\tilde{u}_0$ and u_0.

Hypothesis 3.0.2. The solution $u(t)$ and the initial data u_0 of (2) are both in $H^k(\mathbb{R})$, and are bounded as

$$\|u(t)\|_{H^k} \leq M < \rho \text{ and } \|u_o\|_{H^k} \leq C < \infty, \tag{6}$$

for $0 \leq t \leq T$ and $M < \rho < \infty$.

We define following set of integers such that:

$$s \geq 1, \quad m = s + 3, \quad n = s + 1 = m - 2, \tag{7}$$

We specify for which integers the hypothesis should hold in the lemmas and theorems for the Strang splitting method.

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