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Rotation and modified Ohm's law influence on magneto-thermoelastic micropolar material with microtemperatures

Mohamed I.A. Othman[a,b,*], Ramadan S. Tantawi[a], Mohamed I.M. Hilal[a,c]

[a] Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt
[b] Department of Mathematics, Faculty of Science, Taif University 888, Taif City, Saudi Arabia
[c] Department of Basic Sciences, Faculty of Engineering Sciences, Sinai University, El-Arish, Egypt

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ABSTRACT

The aim of the present article is studying the influence of the rotation on micropolar magneto-thermoelastic material with microtemperatures. The modified Ohm's law insures the temperature gradient and the charge density effects on the governing equations of the studied system. The normal modes method is used to get the solution of the physical quantities of the problem. The comparisons investigated numerically and represented graphically in the presence and the absence of the rotation, the temperature gradient coefficient k_0 and the micropolar effect.

1. Introduction

Modern engineering structures as micropolar materials are often made up of materials possessing an internal structure. The classical theory of elasticity has no an adequate representation of the behavior of the micropolar materials. The analysis of the micropolar materials requires incorporating the theory of oriented media. Eringen [1–4] established the micropolar elasticity to describe the deformation of elastic media with oriented particles. Nowacki [5–7] extended the micropolar theory to extend the thermal effects of micropolar continua. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. The granular media and the multi-molecular bodies whose microstructures act as an evident part in their macroscopic responses are examples of the micropolar materials. The physical nature of these materials needs an asymmetric description of the deformations, while theories for the classical elasticity fail to accurately predict their physical and mechanical behavior. Othman et al. [8] studied the effect of rotation on micropolar thermoelastic material with two temperatures. Some problems of micropolar thermoelasticity are discussed in [9–14]. The study of propagation of plane waves in a rotating media is important in many realistic problems as the rotation bodies in celestial mechanics and heavenly bodies such as the rotation of the moon and the earth. Schoenberg and Censor [15] discussed the propagation of the waves in a rotating elastic medium for any orientation of the rotation axis with respect to free space taking into them study the Coriolis and the Centripetal acceleration. Abbas and Othman [16] studied the effect of rotation on thermoelastic waves in a homogeneous isotropic hollow cylinder.

The theory of microtemperatures deals with the temperature wave propagation in a rigid heat conductor which allows for variation of thermal properties at a microstructure level. The possibility that the suspension might have a different thermal

* Corresponding author at: Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt. Tel.: +20 1121353704.
E-mail addresses: m_i_a_othman@yahoo.com (M.I.A. Othman), ramadan_tantawi1@yahoo.com (R.S. Tantawi), mimhilal@yahoo.com (M.I.M. Hilal).

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microstructure to the carrier fluid should therefore be investigated. However, nanostructures in solids are also important. Cryogenic liquids are heavily involved in space research and such liquids must be stored in stainless steel vessels known as run-tanks. The large thermal stresses placed on the solid vessels may be associated with thermal microstructure effects and hence there is certainly a need for a well-structured theory for a rigid solid which allows for microtemperature effects. Grot [17] established the thermodynamic theory of elastic materials with inner structures contains the microdeformations and the microelements possess microtemperatures. Eringen and Kafadar [18] discussed the basis of the micro-elements which have microtemperatures. Riha [19] presented a study of the heat conduction in materials with inner structures. Iesan and Quintanilla [20] constructed the linear theory of thermoelasticity for materials with inner structure whose particles. Iesan [21,22] presented the mathematical model of the theory of micromorphic elastic solids with microtemperatures since the microelements possess microtemperatures and can stretch and contract independently of their translations. Casas and Quintanilla [23] studied the exponential stability in thermoelasticity with microtemperatures. Scalia and Svandze [24] discussed the solutions of the theory of thermoelasticity with micro-temperatures. Iesan [25] studied the thermoelastic bodies with microstructure and micro-temperatures.

Studying the interaction between magnetic field and stress and strain in a thermo-elastic medium is very remarkable due to its much application in the fields of geophysics, plasma physics, related topics essentially in the nuclear fields, since the extremely high temperatures and the temperature gradients, in addition to the magnetic fields originating inside the nuclear reactors, also for the emissions of electromagnetic radiations from nuclear devices and for understanding the effect of the Earth's magnetic field on seismic waves. Sherief and Helmy [26] investigated 2D problem for a half-space in magneto-thermoelasticity with thermal relaxation. Othman [27] studied the generalized electro-magneto-thermoelastic plane waves by thermal shock problem in a finite conductivity half-space with one relaxation time. Othman et al. [28] investigated the effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation. Recently, Othman and Tantawi [29] discussed the effect of magnetic field on an infinite conducting thermoelastic rotating medium. It was assumed that the interactions between the magnetic field and the electric field take place by means of the Lorentz forces appearing in the equations of the motion and by means of a term entering Ohm's law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field. Ohm's law was modified by the inclusion of the temperature gradient. Ohm's law modification for the temperature gradient stated that the strength of the current at each point is proportional to the gradient of electric potential. The accuracy of the assumptions that flow proportional to the gradient is more readily tested, using modern measurement methods, for the electrical case than for the heat case.

The present study establishes the 2D problem of a rotating, linear, isotropic, homogeneous magneto-micropolar thermoelastic material with microtemperatures. The normal mode method is used to obtain the solutions of the physical quantities of the field quantities. These quantities calculated analytically and numerically then represented graphically to show the exactness of the solutions in the absence and the presence of the physical operators used in the problem.

2. Basic equations

Consider the linear theory of thermodynamics for isotropic elastic materials with inner structure. According to Eringen [3], Isean [25] and Schoenberg and Censor [15], the field equations and the constitutive relations for a rotating, linear, isotropic, homogeneous, micropolar, magneto-thermoelastic material with microtemperatures without body forces, body couples, heat sources and first heat source moment, can be considered as:

$$\sigma_{ij,i} + F_i = \rho\,[\ddot{u}_i + \{\,\Omega \times (\Omega \times u)\}_i + (2\,\Omega \times \dot{u})_i], \tag{1}$$

$$m_{ij,i} + \varepsilon_{ijr}\,\sigma_{ir} - \mu_1\,(\nabla \times\, w)_i = j\rho\,[\ddot{\phi} + \,(\Omega \times \dot{\phi})]_i, \tag{2}$$

$$k_6\,w_{i,jj} + (k_4 + k_5)\,w_{j,ij} + \mu_1\,(\nabla \times\, \dot{\phi})_i - k_2\,w_i - b\,w_{i,t} - k_3\,T_{,i} = 0, \tag{3}$$

$$k\,T_{,ii} - \rho\,C_e\,T_{,t} - \gamma_1\,T_0\,u_{i,it} + k_1\,w_{i,i} = 0, \tag{4}$$

$$\sigma_{ij} = \lambda\,u_{r,r}\,\delta_{ij} + \mu\,(u_{i,j} + u_{j,i}) + k^*\,(u_{j,i} - \varepsilon_{ijr}\,\phi_r) - \gamma_1\,T\,\delta_{ij}, \tag{5}$$

$$m_{ij} = \alpha\,\phi_{r,r}\,\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i}, \tag{6}$$

$$q_i = k\,T_{,i} + k_1\,w_i, \tag{7}$$

$$q_{ij} = -k_4\,w_{k,k}\,\delta_{ij} - k_5\,w_{i,j} - k_6\,w_{j,i}, \tag{8}$$

$$Q_i = (k_1 - k_2)\,w_i + (k - k_3)\,T_{,i}, \tag{9}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{10}$$

$$F_i = \mu_0 (J_i \times H_i). \tag{11}$$