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Optimal harvesting of a stochastic mutualism model with Lévy jumps

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ABSTRACT

In this letter, a stochastic mutualism model with Lévy jumps and harvesting is considered. Under some simple assumptions, sufficient and necessary criteria for the existence of optimal harvesting policy are established. The optimal harvesting effort and the maximum of sustainable yield are also obtained. The effects of random noises on the optimal harvesting of the model are discussed and some numerical simulations are introduced to illustrate the main results.

1. Introduction

One of the most important questions in natural resources management is to establish the ecologically, environmentally, and economically reasonable harvesting policies [1]. Unconstrained harvesting policies and over-harvesting could lead to the extinction of numerous species [2]. Therefore, the investigation on the optimal harvesting policies has important impact on the ecology, environment and the economy [3].

Recently, several authors (see e.g., [4–8]) have concentrated their research work in the field of determining the optimal harvesting policies. Clark [4] obtained the optimal harvesting policy of a logistic population model with catch-per unit effort (CPUE) hypothesis. Fan and Wang [5] established the optimal harvesting policy of a logistic population model with periodic coefficients. Kar [6] considered optimal harvesting policy of a prey–predator model incorporating a prey–refuge and harvesting. Both autonomous and nonautonomous single-species models with either impulsive or continuous harvesting were studied in Braverman and Mamdani [7]. For single-species models with diffusion and harvesting, the optimal harvesting strategies were investigated in Braverman and Braverman [8].

Most of the researchers supposed that all the biological parameters in their models are precisely known and considered the deterministic harvesting models. However, the reality is quite different. In real-world ecosystems, almost all parameters are varying due to both nature and human society, such as cold wave, drought, fire, financial crisis, and so on. Thereby, population dynamics are strongly affected by these environmental perturbations. In fact, many scholars have investigated the optimal harvesting problems of stochastic population systems, see e.g. [9–18]. Under the assumption that the growth rate of the population is affected by the white noise, Beddington and May [9] investigated the following logistic equation with harvesting:

$$dN(t) = N(t)(r - c - aN(t))dt + \alpha N(t)dB(t), \tag{1}$$

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where $c \geq 0$ represents the harvesting effort; $\{B(t)\}_{t\geq 0}$ is a standard Brownian motion defined on a complete probability space $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$; α^2 stands for the intensity of the white noise. Beddington and May [9] showed that if $c < r - 0.5\alpha^2$, then the maximum of expectation of sustainable yield (MESY) is $\max\{Y(c)\} = \max\left\{\lim_{t\to+\infty} \mathbb{E}(cN(t))\right\} = (r - \alpha^2/2)^2/(4a)$ and the optimal harvesting effort (OHE) is $c^* = (r - \alpha^2/2)/2$. Kar [13] got the spectral density of each species for a delay stochastic competition model with harvesting by using the Fourier transform methods. Tran and Yin [3] studied the optimal harvesting strategies for stochastic competitive Lotka–Volterra systems with Markovian switching. Zou et al. [15] considered the optimal harvesting problem of a stochastic Gompertz model by using the ergodic method. Song et al. [16] considered the optimal harvesting problems of stochastic logistic models with jumps. Zou and Wang [17] analyzed the optimal harvesting questions of a stochastic competition model with jumps. Liu [18] established the optimal harvesting strategies of a stochastic predator–prey model with time delay.

On the other hand, mutualism is a usual phenomenon in nature. For example, Breton and Addicott [19] have pointed out that more than 48% of terrestrial plants rely on mycorrhizal relationships with fungi to supply inorganic compounds and trace elements. Zoochory is another example [20]: plant produces food (for example, fleshy fruit, overabundance of seeds) for animals, and animals disperse the seeds. The third example is the relationship between anemones and anemone fish [21]: the anemones supply the fish with protection and the fish protect the anemones against butterflyfish, which eat anemones. Therefore, it is important and interesting to investigate the optimal harvesting policy of stochastic mutualism systems. However, to the best of our knowledge, no result of this aspect has been reported, and the effects stochastic noises on the optimal harvesting of stochastic mutualism systems are still unknown. Motivated by these, in this paper we consider the optimal harvesting policy of a stochastic mutualism system and study the effects stochastic noises on the optimal harvesting of the model.

The rest of this paper is arranged as follows. In Section 2, sufficient and necessary criteria for the existence of optimal harvesting policy are established. Then in Section 3, some examples and simulation figures are introduced to validate the theoretical results. Finally, the conclusions are given in Section 4.

2. Mathematical model

One of the classical frameworks for modeling mutualism interactions is the following Lotka–Volterra mutualism model:

$$\begin{cases} l\dfrac{dN_1(t)}{dt} = N_1(t)[r_1 - a_{11}N_1(t) + a_{12}N_2(t)], \\ l\dfrac{dN_2(t)}{dt} = N_2(t)[r_2 + a_{21}N_1(t) - a_{22}N_2(t)], \end{cases} \tag{2}$$

where $N_1(t)$ and $N_2(t)$ stand for the population size of the first species and the second species, respectively. r_i , a_{ij} $(i, j = 1, 2)$ are all positive constants, and r_i represent the growth rates, a_{ii} denote the co-efficients of intraspecific competition, a_{ij} $(i \neq j)$ measure the effects of species j on the growth of species i. In model (2), $a_{ij} > 0$ $(i \neq j)$, hence each species enhances the growth of the other. We choose model (2) not other mutualism models because model (2) is the most basic and important, and some authors have pointed out that model (2) could describe the reality well in many cases (see e.g. [22]). As matter of fact, in recent years model (2) and its generalised forms have received a great deal of attention owing to its theoretical and practical significance (see e.g. [23–29] and the references therein).

Based on the well-known CPUE hypothesis ([4]), we suppose that the species 1 and 2 are subject to exploitation with harvesting effort rates $c_1 > 0$ and $c_2 > 0$, respectively. Then we obtain the following deterministic mutualism model with harvesting:

$$\begin{cases} l\dfrac{dN_1(t)}{dt} = N_1(t)[r_1 - a_{11}N_1(t) + a_{12}N_2(t)] - c_1N_1(t), \\ l\dfrac{dN_2(t)}{dt} = N_2(t)[r_2 + a_{21}N_1(t) - a_{22}N_2(t)] - c_2N_2(t). \end{cases} \tag{3}$$

Now let us take the environmental perturbations into account. To begin with, let us consider white noise. Recall that r_i stands for the growth rate of species i. In practice we always estimate it by an average value plus errors. In general, by the famous central limit theorem, the error term follows a normal distribution. Therefore, several researchers (see e.g., [9,30,31]) have claimed that for short correlation time, one may replace r_i by $r_i + \alpha_i\dot{B}_i(t)$, where we still use r_i to denote the average growth rate; α_i^2 stands for the intensity of the white noise; $\{\dot{B}_i(t)\}_{t\geq 0}$ is a Gaussian white noise process, namely, $\{B_1(t)\}_{t\geq 0}$ and $\{B_2(t))\}_{t\geq 0}$ are two standard Brownian motions defined on $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, P)$. Then model (3) becomes

$$\begin{cases} ldN_1(t) = N_1(t)[r_1 - c_1 - a_{11}N_1(t) + a_{12}N_2(t)]dt + \alpha_1N_1(t)dB_1(t), \\ ldN_2(t) = N_2(t)[r_2 - c_2 + a_{21}N_1(t) - a_{22}N_2(t)]dt + \alpha_2N_2(t)dB_2(t). \end{cases} \tag{4}$$

Let us now take a further step by taking another type of environmental noise into account, namely the Lévy noise. In the real world, the growth of populations often suffer sudden environmental shocks, such as epidemics, toxic pollutants, floods, earthquakes, and so on. For example, the Sandoz Chemical Accident in 1986 caused a massive mortality of wildlife in the Rhine [32]. The Gulf of Mexico oil spill in 2010 has been “already having a ‘devastating’ effect on marine life in the Gulf” [33]. More examples