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Limit cycles bifurcating from the periodic annulus of the weight-homogeneous polynomial centers of weight-degree 2

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ABSTRACT

We obtain an explicit polynomial whose simple positive real roots provide the limit cycles which bifurcate from the periodic orbits of a family of cubic polynomial differential centers when it is perturbed inside the class of all cubic polynomial differential systems. The family considered is the unique family of weight-homogeneous polynomial differential systems of weight-degree 2 with a center. The computations has been done with the help of the algebraic manipulator Mathematica.

1. Introduction and statement of the main results

The study of the number of limit cycles of a polynomial differential system is mainly motivated by the16th Hilbert's problem stated in 1900. See [9] and [15] for more details.

Actually one of the main goals in the qualitative theory of real planar polynomial differential systems is the determination of their limit cycles. One of the ways to produce limit cycles is perturbing a polynomial differential system which has a center and to study the number of limit cycles which can bifurcate from the periodic orbits of the center, up to first order in the small parameter of the perturbation, see for instance [1,3,8,10,14]. This problem is called for some authors the *weak Hilbert's problem*.

There are many methods to study the maximum number of limit cycles that bifurcate from the periodic annulus of a center, i.e. to study the weak Hilbert's problem. Most of them are based on the Poincaré return map, the Poincaré–Melnikov integrals, the Abelian integrals, and the averaging theory. It is well known that in the plane the last three methods are essentially equivalent. The weak Hilbert's problem has been studied by many authors, see for instance the second part of the book [6] and the hundreds of references quoted there. See also [12].

Here we consider *polynomial differential systems* given by

$$\begin{aligned} \dot{x} &= P(x,y), \\ \dot{y} &= Q(x,y), \end{aligned} \tag{1}$$

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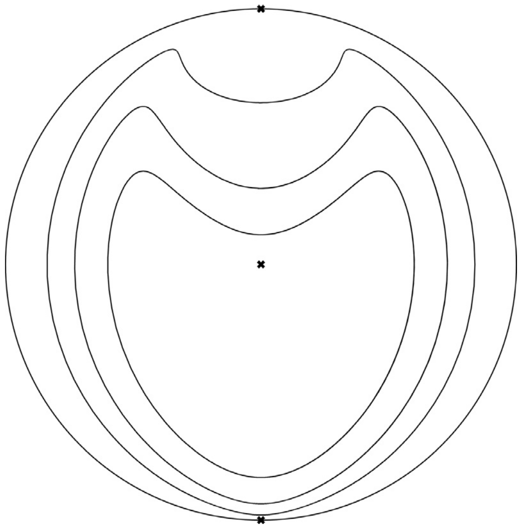

Fig. 1. Phase portrait of the polynomial differential system (2) in the Poincaré disc.

where P and Q are polynomials with real coefficients, the *degree* of the system is the maximum of the degrees of the polynomials P and Q.

System (1) is called *weight-homogeneous* if there exist $(s_1, s_2) \in \mathbb{N}^2$ and $d \in \mathbb{N}$ such that for any $\lambda \in \mathbb{R}^+ = \{\lambda \in \mathbb{R} : \lambda > 0\}$ we have

$$P(\lambda^{s_1}x, \lambda^{s_2}y) = \lambda^{s_1-1+d}P(x, y), \quad Q(\lambda^{s_1}x, \lambda^{s_2}y) = \lambda^{s_2-1+d}Q(x, y).$$

The vector (s_1, s_2) is called the *weight-exponent* of system (1) and d is called *weight-degree* with respect to the weight-exponent (s_1, s_2).

There are few works trying to study the weak Hilbert's problem for weight-homogeneous polynomial differential systems. Our main goal is to solve the weak Hilbert's problem for the weight-homogeneous polynomial differential systems of weight-degree 2.

In [11] the authors classified all centers of a planar *weight-homogeneous* polynomial differential systems up to weight-degree 4. In particular they proved that the unique family of weight-homogeneous polynomial differential systems with a center with weight-degree 2 is

$$\begin{aligned} \dot{x} &= a_{20}x^2 + a_{01}y = P(x, y), \\ \dot{y} &= b_{30}x^3 + b_{11}xy = Q(x, y), \end{aligned} \tag{2}$$

with

$$(b_{11} - 2a_{20})^2 + 8a_{01}b_{30} = -a^2 < 0.$$

The weight-exponent of this family is $(s_1, s_2) = (1, 2)$.

The polynomial differential system (2) has a global center at the origin of coordinates, and its global phase portrait in the Poincaré disc is given in Fig. 1. For more details in order to study the global phase portrait of a polynomial differential system in the Poincaré disc see Chapter 5 of [7].

The main goal of this paper is *to provide an explicit polynomial whose real positive simple zeros gives the exact number of limit cycles which bifurcate, at first order in the perturbation parameter, from the periodic orbits of the center of the weight-homogeneous polynomial differential system (2).*

More precisely we consider the polynomial differential system

$$\begin{aligned} \dot{x} &= a_{20}x^2 + a_{01}y + \varepsilon p(x, y), \\ \dot{y} &= b_{30}x^3 + b_{11}xy + \varepsilon q(x, y), \end{aligned} \tag{3}$$

where

$$\begin{aligned} p(x, y) &= c_{00} + c_{10}x + c_{01}y + c_{20}x^2 + c_{11}xy + c_{02}y^2 + c_{30}x^3 + c_{21}x^2y \\ &\quad + c_{12}xy^2 + c_{03}y^3, \\ q(x, y) &= d_{00} + d_{10}x + d_{01}y + d_{20}x^2 + d_{11}xy + d_{02}y^2 + d_{30}x^3 + d_{21}x^2y \\ &\quad + d_{12}xy^2 + d_{03}y^3, \end{aligned}$$

and ε is a small parameter.