Applied Mathematics and Computation 274 (2016) 119–124

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Extinction in two species nonautonomous nonlinear competitive system

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ARTICLE INFO

MSC:
34D23
92D25
34D20
34D40

Keywords:
Extinction
Nonlinear
Competition
Nonautonomous
Fluctuation lemma

ABSTRACT

A two species nonautonomous nonlinear competitive system is studied in this paper. It is shown that if the coefficients are continuous, bounded above and below by positive constants and satisfy certain inequalities, then one of the components will be driven to extinction while the other one will stabilize at the certain positive solution of a nonlinear single species model. Our result not only generalizing but also improving the result of Li and Chen (2006).

1. Introduction

During the last decade, the study of extinction and permanence of the species has become one of the most important topic in population dynamics ([1–21]). However, most of the study are based on the traditional Lotka–Volterra competitive system [2–21], seldom did scholars consider the nonlinear case [1,22–28].

Recently, Li and Chen [1] studied the extinction property of the following two species competitive system:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)x_2(t) - c_1(t)x_1(t)x_2(t)],\\ \dot{x}_2(t) &= x_2(t)[r_2(t) - a_2(t)x_1(t) - b_2(t)x_2(t) - c_2(t)x_1(t)x_2(t)].\end{aligned} \tag{1.1}$$

where $r_i(t), a_i(t), b_i(t), c_i(t), i = 1, 2$ are assumed to be continuous and bounded above and below by positive constants, and $x_1(t)$, $x_2(t)$ are population density of species x_1 and x_2 at time t, respectively. Given a function $g(t)$, we let g_L and g_M denote $\inf_{-\infty<t<\infty} g(t)$ and $\sup_{-\infty<t<\infty} g(t)$, respectively. Li and Chen [1] showed that if the coefficients of system (1.1) satisfy

$$r_{1L}b_{2L} > r_{2M}b_{1M}, \quad r_{1L}a_{2L} \geq r_{2M}a_{1M} \text{ and } r_{1L}c_{2L} \geq r_{2M}c_{1M}. \tag{1.2}$$

Then second species will be driven to extinction while the first one will stabilize at a certain solution of a logistic equation. It is well known that the growth rate of species need not be always positive, this is realistic since the environment may change on time (e.g. seasonal effects of weather condition, temperature, mating habits and food supplies) and then on some bad time $r_i(t)$ may be negative, in this case, condition (1.2) may no longer hold, and one should develop some new analysis technique to investigate the extinction property of the system (1.1).

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http://dx.doi.org/10.1016/j.amc.2015.10.068

In this paper, we shall investigate the extinction property of the following two species competitive model:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1(t) - a_1(t)x_1^{\alpha_1}(t) - b_1(t)x_2^{\alpha_2}(t) - c_1(t)x_1^{\alpha_1}(t)x_2^{\alpha_2}(t)],\\ \dot{x}_2(t) &= x_2(t)[r_2(t) - a_2(t)x_1^{\alpha_1}(t) - b_2(t)x_2^{\alpha_2}(t) - c_2(t)x_1^{\alpha_1}(t)x_2^{\alpha_2}(t)]\end{aligned} \tag{1.3}$$

together with the initial condition

$$x_i(0) > 0, \quad i = 1, 2. \tag{1.4}$$

Throughout this paper, for system (1.3) we always assume that:

(H) $r_i(t)$, $a_i(t)$, $b_i(t)$ and $c_i(t)$ are continuous bounded functions defined on $[0, +\infty)$; $r_i(t)$, $a_1(t)$ and $b_2(t)$ are bounded above and below by positive constants; $a_i(t) \geq 0$, $b_i(t) \geq 0$, $c_i(t) \geq 0$ for all $t \in [0, +\infty)$; α_i, $i = 1, 2$ are positive constants.

In addition to this section, we arrange the paper as follows: Some basic results are presented in Section 2, the main result are stated and proved in Section 3. In Section 4, an example together with its numeric simulation will be given to illustrate that our result is better than that of Li and Chen [1].

2. Basic results

In this section, we shall develop some preliminary results, which will be used to prove the main result.

Following Lemma 2.1 is a direct corollary of Lemma 2.2 of Chen [23].

Lemma 2.1. *If $a > 0$, $b > 0$ and $\dot{x} \geq x(b - ax^{\alpha})$, where α is a positive constant, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t \to +\infty} x(t) \geq \left(\frac{b}{a}\right)^{1/\alpha}.$$

If $a > 0$, $b > 0$ and $\dot{x} \leq x(b - ax^{\alpha})$, where α is a positive constant, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t \to +\infty} x(t) \leq \left(\frac{b}{a}\right)^{1/\alpha}.$$

Lemma 2.2. *Under the assumption (H) holds, let $x(t) = (x_1(t), x_2(t))^T$ be any positive solution of system (1.3) with initial condition (1.4), then there exists a positive constant M_0 such that*

$$\limsup_{t \to +\infty} x_i(t) \leq M_0, \quad for \ \ all \ \ i = 1, 2,$$

i.e., any positive solution of system (1.3) are ultimately bounded above by some positive constant.

Proof. Let $x(t) = (x_1(t), x_2(t))^T$ be any positive solution of system (1.3) with initial condition (1.4), from the first equation of system (1.3), we have

$$\dot{x}_1(t) \leq x_1(t)[r_1(t) - a_1(t)x_1^{\alpha_1}(t)] \leq x_1(t)[r_{1M} - a_{1L}x_1^{\alpha_1}(t)]. \tag{2.1}$$

By applying Lemma 2.1 to differential inequality (2.1), it follows that

$$\limsup_{t \to +\infty} x_1(t) \leq \left(\frac{r_{1M}}{a_{1L}}\right)^{\frac{1}{\alpha_1}} \stackrel{\text{def}}{=} M_1. \tag{2.2}$$

Similarly to the analysis of (2.1) and (2.2), from the second equation of system (1.3), we have

$$\limsup_{t \to +\infty} x_2(t) \leq \left(\frac{r_{2M}}{b_{2L}}\right)^{\frac{1}{\alpha_2}} \stackrel{\text{def}}{=} M_2. \tag{2.3}$$

Set $M_0 = \max\{M_1, M_2\}$, then the conclusion of Lemma 2.2 follows. This end the proof of Lemma 2.2. $\square$

Lemma 2.3. (*Fluctuation lemma*)*([4, Lemma 4]) Let $x(t)$ be a bounded differentiable function on (α, ∞), then there exist sequences $\tau_n \to \infty, \sigma_n \to \infty$ such that*

$$(a) \quad \dot{x}(\tau_n) \to 0 \ and \ x(\tau_n) \to \limsup_{t \to \infty} x(t) = \overline{x} \ as \ n \to \infty,$$

$$(b) \quad \dot{x}(\sigma_n) \to 0 \ and \ x(\sigma_n) \to \liminf_{t \to \infty} x(t) = \underline{x} \ as \ n \to \infty.$$

For the logistic equation

$$\dot{x}(t) = x(t)\Big(r_1(t) - a_1(t)x_1^{\alpha_1}(t)\Big). \tag{2.4}$$

From Lemma 2.1 of Zhao and Chen [26], we have

Lemma 2.4. *Suppose that $r_1(t)$ and $a_1(t)$ satisfy (H), then any positive solutions of Eq.(2.4) are defined on $[0, +\infty)$, bounded above and below by positive constants and globally attractive.*