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Some properties of the twisted Changhee polynomials and their zeros

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ABSTRACT

Kim et al. [9] studied some special polynomials and numbers which are closely related to Changhee polynomials and numbers, and Park et al. (2014) [17] studied the twisted Changhee polynomials and numbers.

In this paper we consider the twisted Changhee polynomials and numbers. From these polynomials and numbers, we derive some identities. Furthermore, we investigate the higher-order twisted Changhee polynomials and numbers and also discuss some computations of zeros of the twisted Changhee polynomials.

1. Introduction

It is well-known that the Euler numbers defined by the generating function to be

$$\frac{2}{e^t+1} = \sum_{k=0}^{\infty} E_k \frac{t^k}{k!}, \quad \text{(see [2, 4, 5, 7, 12, 15])} \tag{1}$$

When $x = 0$, $E_n = E_n(0)$ are called Euler numbers. As is well-known, the Changhee polynomials are given by the generating function to be

$$\frac{2}{2+t}(1+t)^x = \sum_{k=0}^{\infty} Ch_n(x) \frac{t^n}{n!}, \quad \text{(see [1, 3, 6, 8–10, 13, 14, 16])} \tag{2}$$

In this paper we consider the twisted Changhee polynomials and numbers. From these polynomials and numbers, we derive some identities. Furthermore, we investigate the higher-order twisted Changhee polynomials and numbers and also discuss some computations of zeros of the twisted Changhee polynomials.

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2. The twisted Changhee polynomials and numbers

Let p be an odd prime number. We assume that $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of p-adic integers, the field of p-adic rational numbers and the completion of algebraic closure of $\mathbb{Q}_p$. The p-adic $|\cdot|_p$ is normalized as $|P|_P = \frac{1}{p}$. The fermionic p-adic integral on $\mathbb{Z}_p$ is defined by Kim to be

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-1}(x) = \lim_{N\to\infty}\sum_{x=0}^{p^N-1} f(x)(-1)^x, \quad \text{(see [1–16])} \tag{3}$$

where $f_1(x) = f(x+1)$.

Let $T_p = \cup_{n\geq 1} C_{p^n} = \lim_{n\to\infty} C_{p^n} = C_{p^\infty}$ be the locally constant space, where $C_{p^n} = \{\xi | \xi^{p^n} = 1\}$ is the cyclic group of order p^n. For $\xi \in T_p$, we denote the locally constant function ϕ_ξ by

$$\phi_\xi : \mathbb{Z}_p \longrightarrow \mathbb{C}_p,\ x \longrightarrow \xi^x \quad \text{(see [5, 16, 18])}. \tag{4}$$

If we take $f(x) = \phi_\xi(x)e^{tx}$, then we have

$$\int_{\mathbb{Z}_p} \phi_\xi(x)e^{tx}d\mu_{-1}(x) = \frac{2}{\xi e^t + 1}, \tag{5}$$

and if we take $g(x) = \phi_\xi(x)(1+t)^x$, then we have

$$\int_{\mathbb{Z}_p} \phi_\xi(x)(1+t)^x d\mu_{-1}(x) = \frac{2}{1+\xi+\xi t}. \tag{6}$$

From (5) and (6), we define the twisted Euler polynomials which are given by the generating function to be

$$\frac{2}{\xi e^t + 1}e^{xt} = \sum_{n=0}^{\infty} E_{n,\xi}(x)\frac{t^n}{n!}. \tag{7}$$

When $x = 0$, $E_{n,\xi} = E_{n,\xi}(0)$ are called the twisted Euler numbers and the twisted Changhee polynomials which are given by the generating function to be

$$\frac{2}{1+\xi+\xi t}(1+t)^x = \sum_{n=0}^{\infty} Ch_{n,\xi}(x)\frac{t^n}{n!}. \tag{8}$$

When $x = 0$, $Ch_{n,\xi} = Ch_{n,\xi}(0)$ are called the twisted Changhee numbers (cf. [5,16]). Thus, by (5) and (7), we get

$$\begin{aligned}
\sum_{n=0}^{\infty} E_{n,\xi}(x)\frac{t^n}{n!} &= \frac{2}{\xi e^t + 1}e^{xt} \\
&= \left(\sum_{n=0}^{\infty} E_{n,\xi}\right)\left(\sum_{m=0}^{\infty}\frac{(tx)^m}{m!}\right) \\
&= \sum_{n=0}^{\infty} E_{n,\xi}\sum_{m=0}^{\infty} x^m\frac{t^{n+m}}{n!m!} \\
&= \sum_{n=0}^{\infty} E_{n,\xi}\sum_{l=n}^{\infty} x^{n-l}\frac{t^l}{n!(l-n)!} \\
&= \sum_{m=0}^{\infty}\sum_{n=0}^{m}\binom{m}{n}E_{n,\xi}x^{n-m}\frac{t^m}{m!}.
\end{aligned} \tag{9}$$

From (9), we obtain the following theorem.

Theorem 2.1. *Let m be a nonnegative integer, $\xi \in T_p$, and $x \in \mathbb{C}_p$. Then we have*

$$E_{m,\xi}(x) = \sum_{n=0}^{m}\binom{m}{n}E_{n,\xi}x^{n-m}. \tag{10}$$

From (6) and (8), we get

$$\begin{aligned}
\sum_{n=0}^{\infty} Ch_{n,\xi}(x)\frac{t^n}{n!} &= \frac{2}{1+\xi+\xi t}(1+t)^x \\
&= \left(\sum_{m=0}^{\infty} Ch_{m,\xi}\frac{t^m}{m!}\right)\left(\sum_{l=0}^{\infty}(x)_l\frac{t^l}{l!}\right)
\end{aligned}$$