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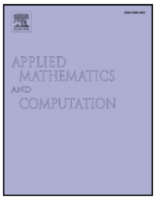

Distributed adaptive control of linear multi-agent systems with event-triggered communications

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ABSTRACT

This paper addresses the average consensus problem of multi-agent systems with general linear dynamics via event-triggered communications. It is shown how the design of both the control law and the trigger functions depends on parameters of the overall system, and an algorithm is proposed to estimate these parameters in a distributed fashion that does not imply an increase in the number of the communications between agents. As a result, it yields an adaptive control law and an update of the trigger functions' parameters only at event times. Proofs of asymptotic convergence to average consensus and existence of positive lower bound for the inter-event intervals are provided. Numerical simulations show the effectiveness of the proposed approach and how it compares to constant values of the parameters.

1. Introduction

In large scale systems, the interconnection of subsystems can be physical or introduced through the control law, such as the case of cooperative control problems in multi-agent systems. One example of a group objective for multi-agent systems is the reaching of a state agreement or consensus, i.e., all agents are supposed to converge to a common state. Such consensus problems have a variety of applications in flocking, attitude synchronization in satellite swarms, distributed sensor networks, congestion control in communication networks, or formation control [1].

A centralized approach to consensus problems makes difficult the scalability of the problem and it is more sensitive to failure or joining of agents, or other external influences, than a neighbor-based coordination strategy. Recent developments in the fields of communication technology, wireless technology, and embedded devices have made possible the implementation of these distributed techniques, since agents are able to exchange information through a shared communication network. As a consequence, the communication network turns to be part of the design problem.

Networked control systems (NCSs) are characterized by the scarcity of communication resources, and in this regard, event-triggered control (ETC) is especially suited for these applications, since it is capable of reducing the amount of communications, while still providing a satisfactory closed-loop performance [2–4]. In ETC, in opposition to time-schedule or periodic control, the control updates are determined by certain events that are triggered depending on the state of the system. Hence, there is a natural interest in applying event-triggering to distributed NCSs.

There are some recent contributions on distributed event-triggered control [5–9], and more specifically, for multi-agent systems [10–14]. The basic idea in all these contributions is that each agent decides when to transmit the measurements based on local information. In the most common implementations, an event is triggered when the error of the system exceeds a tolerable

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bound. However, some of these approaches only consider simple models such as simple or double integrators [10,11], require the continuous monitoring of neighboring agents [10,13,14], or the design of the trigger functions depends on information of the entire network [11,12,14]. This later constraint is very restrictive especially for networks with a large number of agents, since they only has access to nearby nodes [15].

Moreover, global properties such as eigenvalues can also influence the performance by setting the communication links in distributed control for interconnected linear systems [16], determine the maximum tolerable delay in distributed NCSs [17], or serve to estimate the worst case convergence rate of consensus algorithms [18].

There are several algorithms to estimate the matrix eigenvalues in a distributed fashion [19,20]. Most of them apply to hermitian square matrices, and they have been applied, for example, to estimate the algebraic connectivity of a network [21,22]. These algorithms can be also extended to compute the singular values of an arbitrary matrix. Other properties such as the interconnections of physically coupled systems can be computed distributedly with simpler algorithms [23].

In this paper, we propose a distributed event-triggering control approach for multi-agent systems with general linear dynamics to achieve consensus. The proposed design has the following features:

- The consensus is achieved asymptotically.
- The agents only communicate at event times with its neighborhood, broadcasting the state and other parameters that are used in the control algorithm.
- The dependence on properties of the overall system (for instance, eigenvalues) is avoided and replaced by local estimations of those. The estimation is performed online in a completely distributed fashion without increasing the number of communications between agents.
- As a consequence, both the feedback gains of the control law and the trigger functions can change over time, yielding and adaptive control law which is updated only at event-times.
- The existence of a lower bound for the inter-event times is guaranteed.

The rest of the paper is organized as follows: Section 2 briefly reviews some of the mathematical tools that are used through the paper as well as the problem statement. Section 3 discusses the need of an algorithm for distributed parameter estimation and presents the proposed algorithm itself. The main result of the paper is presented in Section 4, where proofs of consensus achievement with the proposed event-triggered control law are provided and the adaptive control law is described. A simulation example is given in Section 5 to illustrate the effectiveness of the proposed approach. Finally, the conclusions and future work are presented in Section 6.

2. Preliminaries

2.1. Matrix analysis

Let $A \in \mathbb{C}^{n\times n}$ be a complex matrix, and let us denote $A^* = (\bar{a}_{ji})$, $\lambda(A) = \{\lambda : \det(A - \lambda I) = 0\}$. The spectral radius $\rho(A)$ is defined as $\rho(A) = \max_i\{|\lambda_i(A)|\}$. For any induced norm $||\cdot||$, it holds that [24]

$$\rho(A) = \lim_{k\to\infty} ||A^k||^{\frac{1}{k}}. \tag{1}$$

We further denote

$$\kappa_A = \|A\|\|A^{-1}\| \quad (0 \notin \lambda(A)), \tag{2}$$

$$\lambda_{max}(A) = \max\{\Re e(\lambda) : \lambda \in \lambda(A)\}, \tag{3}$$

where $\|\cdot\|$ denotes, in this case, the induced 2-norm or spectral norm.

The exponential of A is defined as $e^{At} = \sum_{k=0}^{\infty} \frac{(At)^k}{k!}$.

Let Y be an invertible matrix such that $A = YBY^{-1}$ and $\mu_{max}(A) = \max\{\mu : \mu \in \lambda((A + A^*)/2)\}$. It follows that $\|e^{At}\| = \|Ye^{Bt}Y^{-1}\| \le \kappa_Y e^{\mu_{max}(B)t}$ [25], being κ_Y is defined according to (2). Thus, assume that A is *diagonalizable*, i.e., there exists a matrix D, where $D = \text{diag}(\lambda(A))$, and a matrix V of eigenvectors, such that $A = VDV^{-1}$. Then it holds

$$\|e^{At}\| \le \kappa_V e^{\mu_{max}(D)t} = \kappa_V e^{\lambda_{max}(D)t} = \kappa_V e^{\lambda_{max}(A)t}, \tag{4}$$

where $\lambda_{max}(A)$ is defined according to (3).

For positive-semidefinite matrices, it holds that all the eigenvalues are real and $\lambda_i(A) \ge 0, \quad i = 1, \dots, n,$ or, equivalently $\lambda_n(A) \ge \dots \ge \lambda_2(A) \ge 0$. Moreover, the eigenvectors can always be chosen such that they form an orthonormal basis U, so that $UU^T = I$ and $\|U\| = \|U^T\| = 1$, where U^T is the transpose of U. Then it holds

$$\|A\| = \lambda_n(A) = \rho(A) \text{ and } \|e^{At}\| \le e^{\lambda_n(A)t}. \tag{5}$$

2.2. Algebraic graph theory

Consider and undirected graph $\mathcal{G}$ consisting of N vertices $\mathcal{V}$ and edges $\mathcal{E}$. Two vertices i and j are adjacent if there is an edge $(i, j) \in \mathcal{E}$. For undirected graphs, it holds that $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$. The adjacency matrix $\mathcal{A}$ of a graph is defined by $a_{ij} = 1$ if i