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Explicit group inverse of an innovative patterned matrix

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ABSTRACT

In this paper, we present an innovative patterned matrix, RFPL-Toeplitz matrix, is neither the extension of Toeplitz matrix nor its special case. We show that the group inverse of this new patterned matrix can be represented as the sum of products of lower and upper triangular Toeplitz matrices. First, the explicit expression of the group inverse of an RFPL-Toeplitz matrix is obtained. Second, the decomposition of the group inverse is given. Finally, an example demonstrates availability of the two methods for the group inverse.

1. Introduction

As is well-known, Toeplitz matrix family are also patterned matrix family, have important applications in various disciplines including the elliptic Dirichlet-periodic boundary value problems [1], sinc discretizations of partial and ordinary differential equations [2–7], signal processing [8], numerical analysis [8], system theory [8], etc. Citations of a large number of results have been made in the books of Heining and Rost [9] and of Iohvidov [10]. Mukherjee and Maiti [11] showed that Toeplitz matrices often arise in applications in econometrics, statistics, psychometrics, multichannel filtering, structural engineering, reflection seismology, etc., and it is desirable to have techniques which exploit their special structure. Possible applications of the results related to their determinant, inverse, and eigenvalue problem are suggested.

It is an ideal research area and hot issue for generalized inverses of Toeplitz matrix. In [12], some methods of characterizing and computing generalized inverses of Toeplitz matrices over fields and over rings with the extended Rao condition were presented. In [13], Heinig exhibited that the reflexive generalized inverses of Toeplitz mosaic matrices are Bezoutians. The group inverse of a structured matrix was studied in [14]. In [15], Diao and Wei discussed the structured perturbations of group inverse. In [16], Wei and Diao showed that the group inverse of a real singular Toeplitz matrix can be represented as the sum of products of lower and upper triangular Toeplitz matrices.

In [17], the Moore–Penrose inverse for a matrix bordered by a row and a column was obtained by Hartwig. In [18], Xu presented the Moore–Penrose inverses of Toeplitz matrices can be represented as a sum of products of lower and upper triangular Toeplitz matrices. In [19], Heinig and Hellinger showed that the Moore–Penrose inverses of Hankel matrices were generalized Bezoutians. In [20], based on Bezoutian representations of A^+, the fast algorithms for obtaining the Moore–Penrose inverse of a square Toeplitz matrix A was given by Heinig and Hellinger. In [21], Adukov discussed a generalized inversion of finite rank Hankel operators and Hankel or Toeplitz operators with block matrices having finitely many rows. In [22], Wei modified the algorithm to obtain the Moore–Penrose inverse of a rank-deficient Toeplitz matrix. In [23], the displacement rank of the Drazin inverse was studied by Diao et al. The explicit Drazin inverse of singular Toeplitz matrix was presented in [24]. In [25], the representations for the Drazin inverse of 2×2 block matrices was considered by Li and Wei.

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Definition 1. The row first-plus-last (RFPL) Toeplitz matrix with the first row $(t_0, t_{-1}, t_{-2}, \ldots, t_{2-n}, t_{1-n})$ and the first column $(t_0, t_1, t_2, \ldots, t_{n-2}, t_{n-1})^T$ is meant a square matrix of the form

$$\begin{pmatrix} t_0 & t_{-1} & t_{-2} & \cdots & t_{2-n} & t_{1-n} \\ t_1 & t_0+t_{1-n} & t_{-1} & \ddots & \ddots & t_{2-n} \\ t_2 & t_1+t_{2-n} & t_0+t_{1-n} & \ddots & \ddots & \vdots \\ \vdots & \vdots & t_1+t_{2-n} & \ddots & \ddots & t_{-2} \\ t_{n-2} & t_{n-3}+t_{-2} & \ddots & \ddots & t_0+t_{1-n} & t_{-1} \\ t_{n-1} & t_{n-2}+t_{-1} & t_{n-3}+t_{-2} & \cdots & t_1+t_{2-n} & t_0+t_{1-n} \end{pmatrix}_{n\times n}, \tag{1}$$

denoted by $T_{RFPL}[fr(t_0, t_{-1}, t_{-2}, \ldots, t_{2-n}, t_{1-n}); fc(t_0, t_1, t_2, \ldots, t_{n-2}, t_{n-1})^T]$.

It is particular that the RFPL-Toeplitz matrix is neither the extension of Toeplitz matrix nor its special case and it is a total new patterned matrix. For an RFPL-Toeplitz matrix T_{RFPL} in (1) can be represented as following three splits:

(i)

$$\begin{aligned} T_{RFPL} = {} & \frac{1}{t_0+t_{1-n}} \begin{pmatrix} t_0+t_{1-n} & 0 & \cdots & 0 & 0 \\ t_1+t_{2-n} & t_0+t_{1-n} & \ddots & 0 & 0 \\ t_2+t_{3-n} & t_1+t_{2-n} & t_0+t_{1-n} & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ t_{n-1}+t_0 & t_{n-2}+t_1 & \cdots & t_1+t_{2-n} & t_0+t_{1-n} \end{pmatrix} \\ & \times \begin{pmatrix} t_0+t_{1-n} & t_{-1} & t_{-2} & \cdots & t_{1-n} \\ 0 & t_0+t_{1-n} & t_{-1} & \cdots & t_{2-n} \\ 0 & 0 & t_0+t_{1-n} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & t_{-1} \\ 0 & 0 & 0 & \cdots & t_0+t_{1-n} \end{pmatrix} \\ & - \frac{1}{t_0+t_{1-n}} \begin{pmatrix} t_{1-n} & 0 & \cdots & 0 & 0 \\ t_{2-n} & t_1+t_{2-n} & \ddots & 0 & 0 \\ t_{3-n} & t_2+t_{3-n} & t_1+t_{2-n} & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ t_0 & t_{n-1}+t_0 & \cdots & t_2+t_{3-n} & t_1+t_{2-n} \end{pmatrix} \begin{pmatrix} t_0+t_{1-n} & 0 & 0 & \cdots & 0 \\ 0 & t_{-1} & t_{-2} & \cdots & t_{1-n} \\ 0 & 0 & t_{-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & t_{-2} \\ 0 & 0 & 0 & \cdots & t_{-1} \end{pmatrix}. \end{aligned} \tag{2}$$

(ii)

$$\begin{aligned} T_{RFPL} = {} & \frac{1}{t_0} \begin{pmatrix} t_0 & 0 & \cdots & 0 & 0 \\ t_1 & t_0 & \ddots & 0 & 0 \\ t_2 & t_1 & t_0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ t_{n-1} & t_{n-2} & \cdots & t_1 & t_0 \end{pmatrix} \begin{pmatrix} t_0 & t_{-1} & t_{-2} & \cdots & t_{1-n} \\ 0 & t_0 & t_{-1} & \cdots & t_{2-n} \\ 0 & 0 & t_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & t_{-1} \\ 0 & 0 & 0 & \cdots & t_0 \end{pmatrix} - \frac{1}{t_0} \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ t_1 & 0 & \ddots & 0 & 0 \\ t_2 & t_1 & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ t_{n-1} & t_{n-2} & \cdots & t_1 & 0 \end{pmatrix} \\ & \times \begin{pmatrix} 0 & t_{-1} & t_{-2} & \cdots & t_{1-n} \\ 0 & 0 & t_{-1} & \cdots & t_{2-n} \\ 0 & 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & t_{-1} \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} + \begin{pmatrix} t_{1-n} & 0 & \cdots & 0 & 0 \\ t_{2-n} & t_{1-n} & \ddots & 0 & 0 \\ t_{3-n} & t_{2-n} & t_{1-n} & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ t_0 & t_{-1} & \cdots & t_{2-n} & t_{1-n} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}. \end{aligned} \tag{3}$$