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Axisymmetric shape of gas bubbles in a uniform flow: Numerical study of bifurcation by analytic continuation

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ABSTRACT

This paper investigates axisymmetric shapes of an incompressible inviscid fluid bubble in a uniform flow. Approximate solutions are constructed for the governing nonlinear boundary value problem using the domain perturbation method. Bifurcation and turning points are located by applying a special type of Hermite–Padé approximation.

1. Introduction

The rupture of liquid drops by a gas flow is of great importance in engineering and technology, including spray painting, agriculture (drip irrigation, pesticide drift), atomization etc. In combustion theory, models of fuel droplets are used to describe droplet vaporization in a high-temperature environment in devices such as diesel engines, gas turbines, furnaces and rocket motors [1–3]. The situation of a single drop subjected to aerodynamic forces due to slip velocity between the drop and the surrounding gas can also simulate a raindrop falling through the atmosphere [4].

Since the early 1950′s many theoretical and experimental works has been performed on the study of the deformation of a liquid drop or gas bubble in irrotational flow. The analysis of the forms goes back to Davies and Taylor [5]. They studied air bubbles rising in nitrobenzene or water through measurements of photographs carried out by spark photography. Furthermore they found an analytical expression relating the velocity of rise and the radius of curvature in the region of the vertex.

The parameters that govern the movement of a rising or falling drop or bubble, and consequently characterize its shape, lead through a dimensional analysis to three independent groups of dimensionless numbers: Reynolds, Morton and Weber numbers.

The Weber number represents the ratio between the forces tending to deform the bubble and those which tend to stabilize the interface. Hence, the Weber number indicates whether the kinetic or the surface tension energy is dominant. Theoretical analysis has been directed towards the study of the interaction of the surface tension and hydrodynamic pressure leading to unstable forms, i.e. the existence of a critical Weber number.

This critical value corresponds to a subcritical bifurcation, which means that above this value there is no stable equilibrium position. In this case, due to the growth of capillary wave instabilities, the drop breaks up. This is one of the atomization processes; See [6]. The accurate determination of this critical value is useful for the calibration of nozzles producing calibrated drops.

In the following, we will briefly review some of the previous results and present some new along this line of research.

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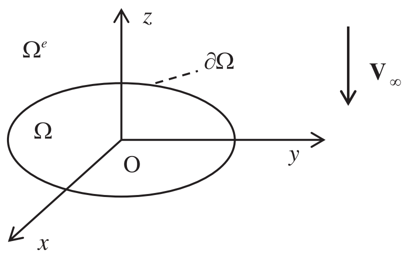

Fig. 1. Schematic profile of the drop.

Saffman [7] examined the rise of small air bubbles and presented both experiments and theory to characterize the bubble's trajectory from a straight line vertical to a zigzag path or to a spiral movement. In the planar zigzag motion, he determined the value of the Weber number at which the rectilinear motion becomes unstable.

Hartunian and Sears [8] investigated both experimentally and theoretically the instability of small gas bubbles moving uniformly in various liquids. By assuming that the surface remains spherical even for a positive Weber number, they found a critical value of the Weber number above which the bubble's shape becomes unstable.

Moore [9,10] considered the rectilinear motion of an oblate ellipsoidal bubble and obtained a first approximation to the drag coefficient which is an essential factor to predict the trajectory of the bubble.

El Sawi [11] examined the same problem using an appropriate approximation based on the virial method. He found that the moving air bubbles in the water are deformed and their forms can be approximated by an ellipsoid of revolution. Analytical expression relating Weber number and the aspect ratio of the bubble was derived, which led to an accurate and close value compared to that obtained by the previous two authors. He also extended his study, taking into account the effect of gravity.

During the last decades, several investigations have been conducted on drop shapes and stability of this type of free boundary problem (see for instance [12–17] and the references therein).

The main purpose of this paper is the investigation of axisymmetric shapes of a deformable bubble in a uniform flow. We use a domain perturbation method which consists in taking as principal unknown a transformation field from the initial known position of the bubble to the unknown position depending on a positive parameter corresponding here to the Weber number. Velocity and pressure fields relating to flow and the shape of the bubble are determined as a power series in terms of the Weber number. Bifurcation and turning points are located by applying a special type of Hermite–Padé approximation. Traditionally this method has been used to determine the location and the nature of the singularity of a function from its power series expansion, as well as its evaluation on its branch cuts. This technique has been expanded by several investigators to find the singularities related to different physical problems.

In the following sections, the problem is formulated, analyzed and discussed.

2. Governing equations

We consider a steady flow caused by rectilinear motion of a bubble or drop with constant velocity $\mathbf{V}_\infty$. Viscosity and gravity are both neglected. We denote by Ω the closed domain occupied by the fluid, by $\partial\Omega$ the boundary of Ω and by $\mathbf{n}$ its outward unit normal vector. The surrounding fluid occupies the domain $\Omega^e = \mathbb{R}^3 \backslash \Omega$. The situation is portrayed in Fig. 1.

The shape of the bubble is governed by the balance between the dynamic of the pressure distribution and the surface tension. The problem will be set in a reference frame Oxyz with the origin attached to the center bubble. We assume without loss of generality, that $\mathbf{V}_\infty$ is parallel to the coordinate axis Oz.

The fluid velocity at infinity is then given by $\mathbf{V}_\infty = -V_\infty \mathbf{e}_z$, where $\mathbf{e}_z$ is a unit vector along axis z. We assume throughout that the flow is incompressible and irrotational. The latter condition implies the existence of a potential function u due to the presence of the bubble, such that the velocity field may be written as $\mathbf{v} = \mathbf{V}_\infty + \nabla u$, with $u \to 0$, and $\nabla u \to 0$ at infinity. The incompressibility condition $\nabla \cdot \mathbf{v} = 0$ and the slip condition at the boundary of the bubble gives the following governing equations for the fluid flow in the unknown domain Ω.

$$\begin{cases} \Delta u = 0 \quad \text{in } \Omega^e & (1.1) \\ \dfrac{\partial u}{\partial n} = -\mathbf{V}_\infty.\mathbf{n} \quad \text{on } \partial\Omega & (1.2) \end{cases}$$

The momentum equations reduce to the Bernoulli equation:

$$\rho \frac{\mathbf{v}^2}{2} + p = \rho \frac{\mathbf{V}_\infty^2}{2} + p_\infty \tag{2}$$

where p is the pressure field, p_∞ is the ambient fluid pressure and ρ is the volume density. We assume that the internal bubble pressure p_0 inside the drop is constant. This assumption is consistent with the bubble experiments of Walters and Davidson [18, 19] showing that the bubble volume remains approximately constant during its evolution. In the presence of surface tension forces, there exists a discontinuity in pressure across the bubble interface with the fluid pressure at the bubble boundary given by the Laplace relation $p = p_0 - \sigma C$, where σ is the surface tension and C the mean curvature of the surface $\partial\Omega$.