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On travelling wave solutions of the diffusive Leslie–Gower model

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ABSTRACT

We investigate the diffusive Leslie–Gower predator–prey model. Travelling wave solutions were found and a minimum wave speed relationship was derived. Linear stability analysis was performed in addition to full numerical simulation of the model. All travelling waves were found to be stable.

1. Introduction

Models that describe predator–prey interactions are well established. These models incorporate competition, cooperation and predation terms to describe realistic dynamics. In these models, features such as extinction of one species or the other, persistence and oscillatory behaviour between species are commonly found. Different formulations of the above mentioned cases are an area of continuing research [2,14,16,21,26].

These models generally consist of two or more ordinary differential equations (ODEs) to represent the interactions of predator and prey. Multiplicity of solutions, limit cycles and chaos are among the features found in these type of systems. For example, Gilpin and Rosenzweig [9] have discussed the effects of enrichment to the predator and prey system that could lead to a limit cycle or periodic solution. Rinaldi et al. [25] studied a classical predator–prey model with a varying environment. They investigated several types of bifurcation properties and found chaos exists in the system. Harrison [10] conducted bacterial population experiments and validated the results using various modified predator–prey models.

Another predator–prey model commonly studied is the Leslie–Gower (LG) model

$$\frac{dX}{dt} = aX - \frac{bX^2}{Y}, \tag{1a}$$

$$\frac{dY}{dt} = cY - dY^2 - eXY, \tag{1b}$$

which is a type of ratio-dependent system. Introduced by Leslie [17] and Leslie and Gower [18], the model assumes that the carrying capacity of the predator X changes proportionally to the prey Y, and that the carrying capacity of the prey is limited by a fixed value c/d. In contrast, the classical models assume that the prey population grows without bound in the absence of the predators [2].

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System (1) has a unique coexistence equilibrium

$$X^* = \frac{ac}{ae+bd}, \quad Y^* = \frac{bc}{ae+bd}.$$

Korobeinikov [13] found a Lyapunov function for the LG model and showed that the equilibrium is globally stable, and a simpler case of LG model was considered by Safuan et al. [29].

Since the LG model has been introduced, numerous studies have modified and investigated the properties of system (1) to model various predator–prey populations. Collings [5] applied several functional forms to system (1) to investigate changes in the dynamical behaviour of mite predator–prey populations. Seo et al. [31] compared a LG-type model with another predator–prey model and found both models exhibit limit cycle solutions. A LG model with the effect of prey refuge was considered by Chen et al. [3]. They found that if the amount of prey refuge is increased, predator and prey densities can either increase or decrease. Chen and Chen [4] investigated a LG model with feedback controls and found that the feedback control variables have no effect on the global stability of the system, but only alter the location of the coexistence equilibria. Modified LG models for three species populations are also available, see references [1,15,27,28].

The aforementioned studies help biologists and ecologists to understand the dynamics that evolve between predators and preys temporally without any spatial dimensions. To incorporate the distribution of populations over time and space domains, spatio-temporal models should be considered. System of partial differential equations (PDEs) will provide more information to explain the population distribution, the wave speed and the effects of diffusivity of each species over a space domain.

The LG model can be extended to include diffusive effects and takes the form

$$\frac{\partial U}{\partial t} = d_1 \frac{\partial^2 U}{\partial x^2} + aU - \frac{bU^2}{V}, \tag{2a}$$

$$\frac{\partial V}{\partial t} = d_2 \frac{\partial^2 V}{\partial x^2} + cV - dV^2 - eUV, \tag{2b}$$

where U is the predator and V is the prey population. Parameters d_1 and d_2 are the diffusion coefficients for the predator and prey, respectively. The predator grows logistically with growth rate a and is limited by the availability of prey with parameter b. The prey also grows logistically with growth rate c. The term eUV represents the effect of predation which reduces the prey's per capita growth rate.

There have been several investigations into the diffusive LG model (2). Du and Hsu [6] investigated the behaviour of steady-state solutions of system (2) in environments that are homogenous and heterogeneous (where the constant parameter b in Eq. (2a) is replaced by a spatial-dependent function $b(x)$). They found that the system has no non-constant positive solution in a homogeneous environment whereas in a heterogeneous environment, a non-constant solution can be obtained. Ko and Ryu [12] investigated non-constant positive steady-states of system (2) with general functional response and found in some conditions, there may have more than one non-constant positive steady-state. A diffusive LG model with a protection zone for the prey was studied by Du et al. [7]. They found results on the asymptotic profile of positive solutions of the model for large intrinsic predator growth rates. A cross-diffusion LG model was considered by Li and Zhang [19]. Depending on the natural and cross-diffusion coefficients, they showed the existence or non-existence of a non-constant positive solution of the system.

In this investigation, we are interested in studying travelling wave solutions within a homogeneous diffusive LG model. The wavefront solution provide information regarding how both populations disperse over space. It is a standard approach to consider travelling waves solution when investigating reaction-diffusion system. For instance, Dunbar [8] investigated a diffusive Lotka–Volterra model that gave rise to a travelling wavefront solution, and simulated the PDE using the method of lines. Huang and Weng [11] applied several numerical methods to study travelling waves for a diffusive predator–prey system with a general functional response. Our aim is to use a similar approach as used by Dunbar to determine and analyse the travelling wave solutions for the diffusive LG model in one spatial dimensional. We also numerically solve the PDEs with different initial conditions.

2. Equilibrium and stability analysis

Introducing the transformations $u = eU/c$, $v = eV/c$, $\tau = at$, and $\xi = \sqrt{a/d_2}\,x$, we arrive at the non-dimensional version of system (2)

$$\frac{\partial u}{\partial \tau} = \delta \frac{\partial^2 u}{\partial \xi^2} + u\left(1 - \frac{\alpha u}{v}\right), \tag{3a}$$

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial \xi^2} + \beta v(1 - \gamma v - u), \tag{3b}$$

where

$$\alpha = b/a, \beta = c/a, \gamma = d/e \text{ and } \delta = d_1/d_2.$$

Let the travelling wave solution have the form $u(\xi,\tau) = u(\zeta)$, and $v(\xi,\tau) = v(\zeta)$ where $\zeta = \xi - s\tau$ is a moving frame with speed s. Substitute these into the system (3) to give a system of second order ODEs which is the travelling wave system

$$\delta u'' + su' + u\left(1 - \frac{\alpha u}{v}\right) = 0, \tag{4a}$$