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Dynamic optimization for robust path planning of horizontal oil wells

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ABSTRACT

This paper considers the three-dimensional path planning problem for horizontal oil wells. The decision variables in this problem are the curvature, tool-face angle and switching points for each turn segment in the path, and the optimization objective is to minimize the path length and target error. The optimal curvatures, tool-face angles and switching points can be readily determined using existing gradient-based dynamic optimization techniques. However, in a real drilling process, the actual curvatures and tool-face angles will inevitably deviate from the planned optimal values, thus causing an unexpected increase in the target error. This is a critical challenge that must be overcome for successful practical implementation. Accordingly, this paper introduces a sensitivity function that measures the rate of change in the target error with respect to the curvature and tool-face angle of each turn segment. Based on the sensitivity function, we propose a new optimization problem in which the switching points are adjusted to minimize target error sensitivity subject to continuous state inequality constraints arising from engineering specifications, and an additional constraint specifying the maximum allowable increase in the path length from the optimal value. Our main result shows that the sensitivity function can be evaluated by solving a set of auxiliary dynamic systems. By combining this result with the well-known time-scaling transformation, we obtain an equivalent transformed problem that can be solved using standard nonlinear programming algorithms. Finally, the paper concludes with a numerical example involving a practical path planning problem for a Ci-16-Cp146 well.

1. Introduction

The path planning problem for oil wells involves designing an optimal drill path, subject to various engineering specifications, to connect given start and end points. There are many papers devoted to this problem (see, for example, Refs. [2,4,6,15]). The path planning problem can, in fact, be formulated as an optimal switching control problem and thereafter solved using standard gradient-based dynamic optimization techniques [9,10]. In particular, the optimal control software packages MISER 3 [5] and Visual MISER [19] are both applicable.

In practical path planning problems, the path usually consists of a combination of turn/straight segments, where the curvature and tool-face angle (both decision variables) are constant in each segment. Although the segment arc lengths can be measured

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and controlled with high precision from the surface, this is not the case for the curvatures and tool-face angles. Indeed, the actual curvatures and tool-face angles will likely deviate from the optimal values during the real drilling process. Consequently, the well trajectory will deviate from the optimal trajectory, potentially leading to a large target error.

We are only aware of two papers (Refs. [7,8]) that consider the issue of target error sensitivity with respect to inaccuracies in the curvatures and tool-face angles. In Ref. [7], the dispersion between the actual path and the optimal path is modeled by a stochastic perturbation in the dynamic system describing the path trajectory. In Ref. [8], the dispersion is instead modeled by a series of stochastic impulsive jumps applied at the turn segment end-points. Both Refs. [7] and [8] apply the Hookes–Jeeves algorithm to solve a series of nonlinear optimization problems generated by realizations of the random variables modeling the dispersion. Thus, the methods proposed in these references require solving multiple optimization problems, not just one. Moreover, the cost and constraint gradients for these optimization problems are not provided in Refs. [7] and [8], and hence fast gradient-based optimization algorithms such as sequential quadratic programming are not applicable.

In any dynamic system, small changes in the system parameters can cause large changes in the system cost. Therefore, when the system contains parameters whose values are uncertain (whether due to model inaccuracies or implementation errors), the sensitivity of the cost with respect to these parameters should be considered [14]. To this end, Ref. [16] proposes a novel optimal control problem in which the control is chosen to minimize the weighted sum of system cost and system sensitivity. This problem can be solved using the control parameterization technique [17], whereby the control function is discretized to yield an approximate optimal parameter selection problem. However, to generate the cost and constraint gradients in the approximate problem (as required to solve the problem using nonlinear programming methods), four dynamic systems must be solved—the state system, the costate system and two other dynamic systems. This contrasts with standard problems in which only the state and costate systems need to be solved. The method proposed in [16] is cumbersome to implement numerically and cannot be automated using the MISER 3 software without significant changes to the internal source code.

Ref. [13] introduces an alternative computational method for solving the same sensitivity optimal control problem considered in Ref. [16]. In this new method, the system sensitivity is computed by solving a set of auxiliary dynamic systems, and the non-standard sensitivity problem is converted into a standard optimal control problem that can be solved using standard techniques. This idea is extended to impulsive systems in [18].

The purpose of this paper is to apply the sensitivity penalization ideas introduced in Refs. [13,16,18] to the path planning problem for drilling three-dimensional horizontal oil wells. Real drilling operations are susceptible to implementation inaccuracies in the segment curvatures and tool-face angles due to the difficulty in measuring these values accurately from the surface. Thus, our goal is to minimize the target error sensitivity with respect to the curvature and tool-face angle of each turn segment, subject to a constraint specifying the maximum path length compared with the theoretical minimum value. Note that the sensitivity optimal control problems in Refs. [13,16,18] do not consider state constraints, but such constraints are present in the path planning problem due to practical requirements on the path. The computational algorithm we develop is capable of handling these constraints.

The remainder of this paper is organized as follows. In Section 2, both the traditional path planning problem and the new sensitivity problem are defined mathematically. In Section 3, a computational method is developed for evaluating the target error sensitivity. Then, in Section 4, this method is combined with a novel transformation procedure to convert the sensitivity problem into a tractable form. Gradient formulas are derived in Section 5. Simulation results for a real Ci-16-Cp146 well are presented in Section 6. Finally, some concluding remarks are given in Section 7.

2. Problem formulation

2.1. Path planning problem

Consider the three-dimensional path planning problem for horizontal oil wells described in Ref. [2]. Let s be an independent variable representing the distance along the path. Furthermore, let $\boldsymbol{x}(s) = (x_1(s), \ldots, x_5(s))^\top$ be the state vector whose components represent, respectively, the inclination, azimuth, and Cartesian coordinates of the point located at distance s along the path.

The model is based on the following hypotheses.

(H_1). The path is a combination of n smooth turn segments.
(H_2). The curvature and tool-face angle are constant in each turn segment.
(H_3). The path contains no vertical segments.

Let τ_i, $i = 0, 1, \ldots, n$, denote the switching points at which the path changes from one turn segment to another, where $\tau_0 = 0$ corresponds to the path's origin, and τ_n corresponds to the path's terminus. Under the above hypotheses, the dynamic system describing the path is

$$\dot{\boldsymbol{x}}(s) = \boldsymbol{f}(\boldsymbol{x}(s), \boldsymbol{\xi}^i), \quad s \in [\tau_{i-1}, \tau_i), \quad i = 1, 2, \ldots, n, \tag{1}$$

with the initial and intermediate conditions:

$$\boldsymbol{x}(\tau_i) = \begin{cases} \boldsymbol{x}^0, & \text{if } i = 0, \\ \boldsymbol{x}(\tau_i -), & \text{if } i = 1, 2, \ldots, n, \end{cases} \tag{2}$$