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# Comparison of several families of optimal eighth order methods

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ABSTRACT

Several families of optimal eighth order methods to find simple roots are compared to the best known eighth order method due to Wang and Liu (2010). We have tried to improve their performance by choosing the free parameters of each family using two different criteria.

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## 1. Introduction

There is a vast literature on the solution of nonlinear equations, see for example Ostrowski [1], Traub [2], Neta [3] and Petković et al. [4].

Lotfi et al. [5] have developed an eighth order family of optimal methods (denoted LSSS)

$$\begin{aligned} y_n &= x_n - v_n, \\ z_n &= y_n - v_n \frac{t_n}{1-2t_n}, \\ x_{n+1} &= z_n - \frac{f(z_n)}{f'(x_n)} \frac{H(t_n)+K(s_n)}{G(u_n)}, \end{aligned} \tag{1}$$

where from here on we use the following:

$$v_n = \frac{f(x_n)}{f'(x_n)}, \tag{2}$$

and

$$t_n = \frac{f(y_n)}{f(x_n)}, \tag{3}$$

$$s_n = \frac{f(z_n)}{f(x_n)}, \tag{4}$$

$$u_n = \frac{f(z_n)}{f(y_n)}. \tag{5}$$

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The weight functions $H, K, G$ satisfy

$$G(0) = 1, \quad G'(0) = -1, \tag{6}$$

$$K(0) = 0, \quad K'(0) = 2, \tag{7}$$

$$H(0) = 1, \quad H'(0) = 2, \quad H''(0) = 10, \quad H'''(0) = 72. \tag{8}$$

They also include several methods and the following families of methods in their comparative study [5]:

- Sharma2, a family of methods by Sharma and Sharma [6]

$$\begin{aligned} y_n &= x_n - v_n, \\ z_n &= y_n - \frac{f(y_n)}{f'(x_n)} \frac{1}{1-2t_n}, \\ x_{n+1} &= z_n - W(s_n) \frac{f(z_n) f[x_n, y_n]}{f[x_n, z_n] f[y_n, z_n]}, \end{aligned} \tag{9}$$

  with weight function

$$W(s_n) = 1 + \frac{s_n}{1 + \alpha s_n}, \tag{10}$$

  and $\alpha$ is some real parameter. Sharma and Sharma [6] have used $\alpha = 1$.
- CTV, a three-parameter family of methods by Cordero et al. [7]

$$\begin{aligned} y_n &= x_n - v_n, \\ z_n &= y_n - \frac{f(y_n)}{f'(x_n)} \frac{1}{1-2t_n}, \\ x_{n+1} &= w_n - \frac{f(z_n)}{f'(x_n)} \frac{3(\beta_2 + \beta_3)(w_n - z_n)}{\beta_1 (w_n - z_n) + \beta_2 (y_n - x_n) + \beta_3 (z_n - x_n)}, \end{aligned} \tag{11}$$

  where

$$w_n = z_n - \frac{f(z_n)}{f'(x_n)} \left( \frac{1-t_n}{1-2t_n} + \frac{1}{2} \frac{u_n}{1-2u_n} \right)^2, \tag{12}$$

  and $\beta_1, \beta_2$, and $\beta_3$ are real parameters with $\beta_2 + \beta_3 \neq 0$.
  Remark: Cordero et al. [7] have used $\beta_1 = \beta_3 = 0$ and $\beta_2 = 1$.
- CL, a two-parameter family of methods by Chun and Lee [8]

$$\begin{aligned} y_n &= x_n - v_n, \\ z_n &= y_n - \frac{f(y_n)}{f'(x_n)} \frac{1}{(1-t_n)^2}, \\ x_{n+1} &= z_n - \frac{f(z_n)}{f'(x_n)} \frac{1}{(1 - H(t_n) - J(s_n) - P(u_n))^2}, \end{aligned} \tag{13}$$

  where the weight functions should satisfy the following conditions to guarantee eighth order:

$$H(0) = 0, \quad H'(0) = 1, \quad H''(0) = 1, \quad H'''(0) = -3, \tag{14}$$

$$J(0) = 0, \; J'(0) = \frac{1}{2}, \quad P(0) = 0, \quad P'(0) = \frac{1}{2}. \tag{15}$$

  Remark: Chun and Lee [8] have used the following weight functions

$$\begin{aligned} H(t_n) &= -\beta - \gamma + t_n + t_n^2/2 - t_n^3/2, \\ J(s_n) &= \beta + s_n/2, \\ P(u_n) &= \gamma + u_n/2, \end{aligned} \tag{16}$$

  and $\beta$ and $\gamma$ are real parameters chosen to be zero for simplicity.

In our previous work, we found that it is better not to use polynomials as weight functions, therefore we will use the following:

$$\begin{aligned} J(t) &= \frac{a_1 + b_1 t}{1 + \delta_1 t}, \\ P(t) &= \frac{a_2 + b_2 t}{1 + \delta_2 t}, \end{aligned}$$