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# Assessment of a multi-state system under a shock model

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ABSTRACT

A system is subject to random shocks over time. Let $c_1$ and $c_2$ be two critical levels such that $c_1 < c_2$. A shock with a magnitude between $c_1$ and $c_2$ has a partial damage on the system, and the system transits into a lower partially working state upon the occurrence of each shock in $(c_1, c_2)$. A shock with a magnitude above $c_2$ has a catastrophic affect on the system and it causes a complete failure. Such a shock model creates a multi-state system having random number of states. The lifetime, the time spent by the system in a perfect functioning state, and the total time spent by the system in partially working states are defined and their survival functions are derived when the interarrival times between successive shocks follow phase-type distribution.



## 1. Introduction

In shock models, the failure of a system is usually based on the time between two successive shocks or the damage caused by shock(s). So far in the literature, various shock models have been defined and studied. The shock models can be classified in five groups: cumulative shock model [6], extreme shock model [17], run shock model [14], $\delta$-shock model [4,11], and mixed shock model. The mixed shock model is obtained by combining two different shock models. For example, in the model introduced by Gut [7], the system breaks down when the cumulative shocks reach some "high" level or when a single "large" shock appears, whichever comes first. This is the combination of the cumulative and extreme shock models. Other types of mixed shock models have been studied in [9,14,19].

According to the well-known extreme shock model, the system fails when the magnitude of an individual shock exceeds some given level $c$ [17]. If $X_i$ and $Y_i$ represent respectively the time between the $(i-1)$st and $i$th shocks, and the magnitude of the $i$th shock, $i \geq 1$, then the lifetime of the system under the extreme shock model is defined as $T = \sum_{i=1}^{W} X_i$, where the stopping random variable $W$ is defined by $\{W = w\} \equiv \{Y_1 \leq c, \ldots, Y_{w-1} \leq c, Y_w > c\}$. Extreme shock model has been studied in several papers including [2,3,8,20].

In this paper, we introduce an extreme shock model which creates a multi-state system. The system completely fails upon the occurrence of a large shock as in the classical extreme shock model. However, if the magnitude of a shock varies between two critical points, then the system transits into a lower partially working state and still functions with a reduced capacity.

Much attention has been given to study multi-state systems due to their wide applications in many areas such as engineering reliability, population dynamics, game theoretical models, and medicine [1,12,13,16]. Multi-state systems are common in network growth models, where the Matthew effect is present [15,18]. Although the shock models have been widely studied in a binary setting, to the best of our knowledge, their extension in a multi-state setup has not been considered yet. This paper extends the extreme shock model from binary setting to the multi-state case by the help of two critical thresholds.

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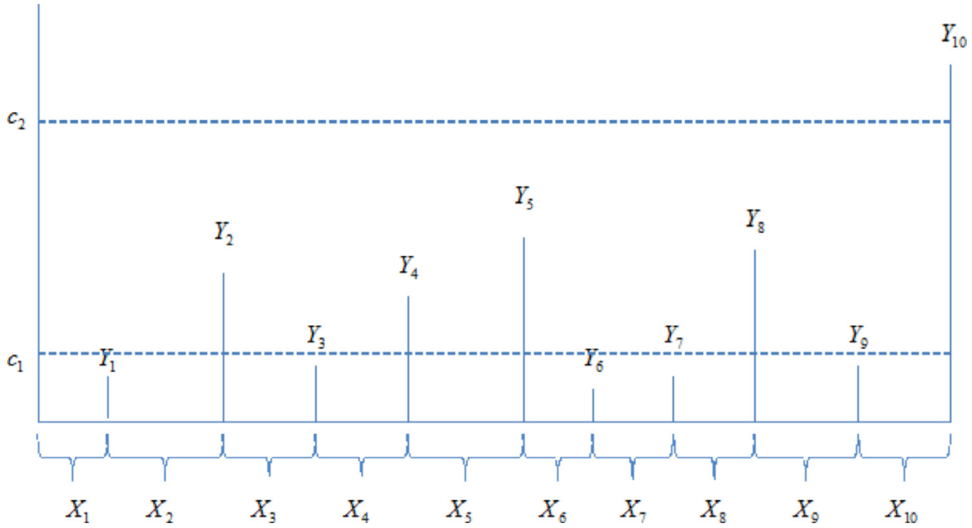


**Fig. 1.** A possible realization of the system.

The present paper is organized as follows. In Section 2, we describe the system with its potential applications. Section 3 involves a detailed analysis on some dynamic reliability properties of the system.

## 2. Description of the system

Let $c_1$ and $c_2$ be two critical levels such that $c_1 < c_2$. A shock with a magnitude between $c_1$ and $c_2$ has a partial damage on the system, and the system transits into a lower partially working state upon the occurrence of each shock in $(c_1, c_2)$. That is, after each shock with magnitude between $c_1$ and $c_2$, the system state is decreasing on one unit. A shock with a magnitude above $c_2$ has a catastrophic effect on the system and it causes a complete failure. Such a system has a random number of states. That is, if $\phi(t)$ denotes the state of the system at time $t$, then $\phi(t) \in \{0, 1, \ldots, M+1\}$, where $M$ is a random variable which represents the number of shocks above $c_1$ and below $c_2$ until the extreme shock above $c_2$. The states "0" and "$M+1$" represent respectively complete failure and perfect functioning states, and there are in total $M+1$ working states which are $\{1, \ldots, M+1\}$. The system is in a perfect functioning state at time $t = 0$, and it stays there until the first shock in $(c_1, c_2)$ or above $c_2$ whichever occurs first.

Let $T^i$ denote the time spent by the system in state $i$, $i = 1, \ldots, M+1$. Clearly, the random variables $T^{M+1}$ and $T = T^1 + \ldots + T^{M+1}$ respectively define the time spent by the system in a perfect functioning state and the time until failure of the system. By the definition,

$$T^{M+1} = \begin{cases} T, & \text{if } M = 0 \\ T^{M+1}, & \text{if } M > 0. \end{cases}$$

The random variable $T - T^{M+1}$ is the time elapsed after the first shock in $(c_1, c_2)$ until the complete failure of the system, or equivalently it is the total time spent by the system in partially working states. This paper is mainly concerned with the derivation of distributions of the random variables $T$, $T^{M+1}$ and $T - T^{M+1}$.

For better understanding the model, two figures are given below. Fig. 1 gives a possible realization of the system. Fig. 2 gives the states of the system for the realization in Fig. 1. Because the number of shocks above $c_1$ and below $c_2$ until the extreme shock above $c_2$ is $M = 4$ for the realization in Fig. 1, the initial state in Fig. 2 is $M + 1 = 5$.

Such a model might be useful for the reliability evaluation of power supply. Power supply life is affected by various stresses such as thermal, mechanical, and electrical. Thus a shock can be considered as one of these stresses or combination of them. When the power supply system is subject to a stress (shock) above a critical level (say $c_1$) it still continues to operate, but at a reduced capacity. For example, static thermal stress degrades components and their basic materials. Bulk capacitors may begin to dry out and resistor coatings may begin to deteriorate. A complete failure of the power supply occurs when the stress (shock) exceeds the level $c_2$ . The random variable $T^{M+1}$ represents the time that the power supply has worked with full capacity.

The model may also be useful in healthcare management. Prostate-specific antigen (PSA) level is an important quantity in active surveillance and postoperative monitoring of prostate cancer patients. That is seriously considered and measured after prostatectomy (surgical removal of the prostate) in follow-up visits. After radical prostatectomy, a PSA of 0.2 ng/ml (nanograms per milliliter) may signal a recurrence of cancer and salvage radiotherapy may be recommended [5]. It should be noted that the cutoff value 0.2 is not fixed, and several studies have used different values. If the PSA increases to >4 ng/ml, for example, there is a chance the tumor may have become locally advanced/advanced, at which point hormone therapy is the only option [5]. The model proposed in this paper can be used in monitoring PSA levels after surgical treatment of a prostate cancer patient. The system (patient) is assumed to be in perfect state after prostatectomy at time $t = 0$. Let the random variables $Y_1, Y_2, \ldots$ represent