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A sixth order transformation method for finding multiple roots of nonlinear equations and basin attractors for various methods

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ABSTRACT

In this contribution, a sixth-order transformation method is proposed and analyzed for finding multiple roots of nonlinear equations, when the multiplicity of the root is not known explicitly. The proposed method does not require the evaluation of second derivative. The basins of attraction of existing transformation methods and the proposed method are presented to demonstrate their performance.

1. Introduction

In this study, we apply iterative methods to find a multiple root α of multiplicity $m > 1$, i.e. $f^{(j)}(\alpha) = 0,\ j = 0, 1, \ldots m-1$ and $f^{(m)}(\alpha) \neq 0$, of a nonlinear equation $f(x) = 0$, where $f(x)$ be the continuously differentiable real or complex function. Modified Newton method [1] is an important and basic method for finding multiple roots

$$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}, \tag{1}$$

which converges quadratically and requires the knowledge of multiplicity m of root α.

In order to improve the order of convergence of (1), several higher-order methods have been proposed in the literature with known multiplicity m, for example, [2–19]. On the other hand, if multiplicity m is not known explicitly, Traub [6] suggested a simple transformation:

$$F(x) = \begin{cases} \dfrac{f(x)}{f'(x)} & \text{if } f(x) \neq 0, \\ 0 & \text{if } f(x) = 0, \end{cases} \tag{2}$$

to find a multiple root of $f(x) = 0$, thereby reducing the task of finding a multiple root to that of solving a simple root of the transformed equation $F(x) = 0$. Thus any iterative method can be used to preserve the original order of convergence. However, using this transformation in Newton method requires the use of $f'(x)$ and $f''(x)$. In order to avoid the calculations of these derivatives, King [20] proposed the secant method, with unknown multiplicity for finding multiple roots of nonlinear equation, which

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used another transformation:

$$F(x) = \frac{-f^2(x)}{f(x - f(x)) - f(x)}. \tag{3}$$

The secant method thus obtained has order of convergence 1.618.

Using the same transformation (3), Iyengar and Jain [21] developed two iterative methods of order three and four for finding multiple roots of nonlinear equations. The third order method is given as:

$$x_{k+1} = x_k - l_1 - l_2, \tag{4}$$

where

$$l_1 = \frac{F(x_k)}{G(x_k)}, \quad l_2 = \frac{F(x_k - l_1)}{G(x_k)}, \quad G(x_k) = \frac{F(x_k + \beta F(x_k)) - F(x_k)}{\beta F(x_k)}. \tag{5}$$

and fourth order method is expressed as:

$$x_{k+1} = x_k - l_1 - l_2 - l_3, \tag{6}$$

where l_1 and l_2 are as defined in (5) and $l_3 = \frac{F(x_k - l_1 - l_2)}{G(x_k)}$.

With the same transformation (3), Wu and Fu [22] developed a quadratically convergent iterative method for multiple roots given as:

$$x_{k+1} = x_k - \frac{F^2(x_k)}{p.F^2(x_k) + F(x_k) - F(x_k - F(x_k))}, \tag{7}$$

where $p \in \mathbb{R}$, $|p| < \infty$.

Wu et al. [23] suggested another transformation:

$$F(x) = \begin{cases} \dfrac{\operatorname{sign}(f(x)) f(x) |f(x)|^{1/m}}{\operatorname{sign}(f(x + \operatorname{sign}(f(x))|f(x)|^{1/m}) - f(x)) f(x) |f(x)|^{1/m} + f(x + \operatorname{sign}(f(x))|f(x)|^{1/m}) - f(x)} & \text{if } f(x) \neq 0, \\ 0 & \text{if } f(x) = 0, \end{cases}$$

and proposed a quadratically convergent method by applying this transformation to modified Steffensen's method [24,25].

Parida and Gupta [26] proposed another transformation:

$$F(x) = \begin{cases} \dfrac{f^2(x)}{\operatorname{sign}(f(x + f(x)) - f(x)) f^2(x) + f(x + f(x)) - f(x)} & \text{if } f(x) \neq 0, \\ 0 & \text{if } f(x) = 0, \end{cases}$$

and obtained a quadratically convergent iterative method given as:

$$x_{k+1} = x_k - \frac{F^2(x_k)}{p.F^2(x_k) + F(x_k) - F(x_k - F(x_k))}, \tag{8}$$

where the parameter p should be chosen such that the denominator is largest in magnitude.

Yun [27] also suggested another transformation for finding multiple root $\alpha \in (a, b)$ of $f(x) = 0$ given as:

$$F(x) = \frac{\epsilon f^2(x)}{f(x + \epsilon f(x)) - f(x)},$$

where ϵ is such that $\max_{a \le x \le b} |\epsilon f(x)| = \delta$. Using this transformation, Yun proposed a quadratically convergent iterative method as follows:

$$x_{k+1} = x_k - \frac{2(x_k - x_{k-1}) F(x_k)}{F(2x_k - x_{k-1}) - F(x_{k-1})}. \tag{9}$$

Recently, by using the transformation (2), Li et al. [28] proposed a fifth-order iterative method for multiple roots of the nonlinear equation $f(x) = 0$, which is given as,

$$\begin{cases} y_k = x_k - \dfrac{F^2(x_k)}{F(x_k + F(x_k)) - F(x_k)}, \\ z_k = y_k - \dfrac{F(y_k) F(x_k)}{F(x_k + F(x_k)) - F(x_k)}, \\ x_{k+1} = z_k - \dfrac{F(z_k)}{F[z_k, y_k] + F[z_k, x_k, x_k](z_k - y_k)}, \end{cases} \tag{10}$$

where $F[.,.]$ and $F[.,.,.]$ are divided differences of F of order one and two respectively.

In this work, our aim is to develop an iterative method of higher order for finding multiple roots of nonlinear equations with unknown multiplicity m. We propose a new modification of Newton method based on the transformation (2) given by Traub [6].