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Distribution of the zero-effort miss distance estimation error in interception problems

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ABSTRACT

In interception problems subject to noise corrupted measurements and random bounded target maneuvers, the miss distance is a random variable with an a-priori unknown distribution. In previous works, such a miss distance distribution could be obtained by a computational approach, assuming that the distribution of the zero-effort miss distance estimation error is known as a function of time. In this paper, this assumption is replaced by an analytical approach based on the knowledge of the measurement error distribution, the estimator structure and the (possibly non-linear) guidance law. For this purpose the shaping filter technique is used, considering that the excitation noise due to the random target maneuvers, as well as the observation noise, are Gaussian white noises. This analytical approach is illustrated by numerical examples.

1. Introduction

Various real life control problems, such as navigation and interceptor guidance, can be formulated as a problem of transferring a linear system, in the presence of noise corrupted state measurements and unknown bounded disturbances, to a prescribed target set in the state space at a prescribed time, by bounded control [1]. In many cases, such a problem can be transformed by a scalarizing transformation [2] to a problem of robust transferring to a point (final time, zero) in the (time, state) plane.

Assuming perfect state information, several classes of deterministic feedback control strategies that robustly transfer a scalar system from some domain of initial positions to a point (final time, zero) are known. The family of robust transferring strategies includes various linear, saturated linear and nonlinear strategies [3] and [4] as well as a differential game based bang–bang strategy [5].

In most real life applications, the state information is corrupted by measurement noise, and not all of the state variables can be directly measured. These facts lead to a significant deterioration in the performance of the *theoretically* robust transferring strategies. Thus, an estimator, restoring and filtering the state variables, becomes an indispensable component of the control loop. Due to the noisy measurements and the uncertain disturbance, the control function receives, instead of the accurate state value, a *random* estimator output. Consequently, the terminal state value also becomes a random variable with an a-priori unknown probability distribution. In order to appreciate the performance deterioration of a deterministic robust transferring strategy by using such stochastic data, the probability distribution of the terminal state value has to be obtained.

In current practice, such a probability distribution is obtained (for any given system dynamics, estimator/control strategy combination, specified disturbance and noise models) by a large set of Monte Carlo simulations [6] and [7]. Although such an

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Nomenclature

a_e^{max}	bound of the target lateral acceleration;
a_p^{max}	bound of the interceptor lateral acceleration;
d_0	scalar in the description of the target set;
$F_{z(t)}(x)$	distribution function of the zero-effort miss distance at the moment of time $t \in [0, t_f)$;
$F_{\lvert z_{t_f}\rvert}(x)$	distribution function of the miss distance;
$F_{\eta(t)}(x)$	distribution function of the zero-effort miss distance estimation error at the moment of time $t \in [0, t_f)$;
$\mathbf{k}(t)$	estimator gain vector;
$\mathbf{P}(t_1, t_2)$	covariance matrix of the reconstruction error;
t	time, s;
t_0	initial time, s;
t_f	time of flight, s;
$u(t)$	interceptor control function;
V_e	target velocity, m/s;
V_p	interceptor velocity, m/s;
$\mathbf{V}(t)$	white noise intensity;
$w_1(t)$	white noise representing the target lateral acceleration;
$w_2(t)$	observation (measurement) white noise;
$\mathbf{x}_0$	initial vector-state of the system;
$\mathbf{x}(t)$	vector-state of the system driven by white noise;
$\mathbf{X}(t)$	vector-state of the actual system with random disturbance (target maneuver);
$x_1(t) = X_1(t)$	separation between the *interceptor* and the *target* normal to the initial line of sight, m;
$x_1(t_f)$	miss distance, m;
$x_2(t) = X_2(t)$	relative velocity normal to the initial line of sight, m/s;
$X_3(t)$	lateral acceleration of the target normal to the initial line of sight, m/s^2;
$x_3(t)$	the 3-rd vector-state component in the system, driven by white noise, m/s^2;
$X_4(t)$	lateral acceleration of the interceptor normal to the initial line of sight, m/s^2;
$\hat{\mathbf{x}}(t)$	estimated vector- state;
$\tilde{\mathbf{x}}(t)$	estimation error;
$y(t)$	observation (line of sight angle), rad;
$\mathbf{z}(t)$	zero-effort miss distance, m;
α	reciprocal of the random lateral target acceleration time constant, s^{-1};
τ_s	time constant of the shaping filter, s;
τ_p	interceptor time constant, s;
σ_a	standard deviation of the random target acceleration;
σ_1	standard deviation of the white noise used in the shaping filter to model the unknown disturbance that acts upon the system;
σ_2	standard deviation of the observation noise;
$\eta(t)$	zero-effort miss distance estimation error;

a-posteriori test is absolutely necessary for validation purpose, it is not useful for an insightful control system design. There is a need for an analytical a-priori estimate of the system performance as a part of an integrated control system design.

In recent previous works dealing with this problem, the system dynamics was modeled either by a discrete [8] or a continuous time [9], scalar linear equation controlled by a saturated linear transferring control strategy. Assuming that the probability distributions of initial state value and the measurement noise are given and the estimation error of the state is known as a function of time, a recurrence formula for the probability distribution of the terminal state value was obtained for the discrete time case [8] while, for continuous time [9], a partial differential equation was derived.

Since in real life the distribution of the zero-effort miss distance estimation error is not known, there is a need to develop an analytical approach for computing this missing element. The objective of the present paper is to outline this approach. By using the so-called *shaping filter* technique [10], the disturbance (a random input to the system) is replaced by the output of a linear *shaping filter* exited by a white noise input.

The structure of the paper is the following: in the next section the stochastic control problem of the interception, subject to noise corrupted measurement and random bounded target maneuvers, is stated. It is followed by determining the stochastic model, suitable for use, where the *shaping filter* is applied. The next section is devoted to the construction of a linear state vector estimator, optimal in the sense of unbiasedness and minimal variance. This estimator is used to find the distribution of the state estimation error as a function of time, based on solving the Riccati equation numerically. Introducing this result into the partial differential equation derived in [9] and solving this equation yields the needed distribution of the terminal state.

The last section presents illustrative numerical examples.