Applied Mathematics and Computation 269 (2015) 308–325

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Turbulent transition modeling through mechanical considerations

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ARTICLE INFO

Keywords:
Transition model
ERCOFTAC
V-model
Spalart–Allmaras
OpenFoam

ABSTRACT

A new transition turbulence model is proposed. The model is based on the computation of the pre-transitional $\overline{u'v'}$ due to mean flow shear. The proposed transition model is fully described and verified against standard benchmark test cases. A detailed analysis of its behavior is performed using the ERCOFTAC flat-plate T3A, T3B and T3C test cases. Further, the model is validated in terms of the pre-transitional computed $\overline{u'v'}$ in comparison to experiments of ERCOFTAC database. The model presents very good results for bypass transition for flows with and without pressure gradient.

1. Introduction

This work is the result of a continuous research effort on turbulence transition models development, [1–3] and [4].

Turbulent flow has always been distinguishable from its laminar state. Transition can be identified as an increase in skin-friction and velocity profile departure from a Blasius distribution. There used to be a certainty about the fact that only turbulent flow had velocity fluctuations. Laminar flow was considered to develop in a stationary fashion. This all changed with the work of Dryden, [5]. This was considered to be the first attempt to successfully measure velocity fluctuations in the laminar flow region. It was concluded, in [5], that it is impossible to perceive a distinction between laminar and turbulent flow solely based upon velocity fluctuation measurements. These fluctuations in the laminar flow region were attributed to the presence of a turbulent free-stream. Later on, the experimental work in [6] also confirmed the presence of laminar fluctuations just before turbulence transition onset. To the best of our knowledge, and according to the award winning paper of Mayle and Schulz, [7], the work of Lin, [8] was the first to analytically evaluate the effects of laminar fluctuations over laminar velocity profiles. This study confirmed the possibility of having velocity fluctuations in a laminar flow, even when maintaining a Blasius velocity profile distribution. In the work of [7] the LKE theory, or Laminar-Kinetic-Energy, was proposed. Following this development, a new tool for transition modeling was available. The first transition models based on LKE were developed by Lardeau et al., [9] and Walters and Leylek, [10]. Others followed this trend, such as Vlahostergios et al., [11]. In the publication of [9], it is stated that, before the increase of skin-friction due to transition, a growth of fluctuation intensity persists in the upper to medium regions of the laminar boundary layer. Although these oscillations have their origin in the free-stream turbulence, they do not have the same known fully turbulent ratio of $\frac{-\overline{u'v'}}{k} \approx 0.3$. Instead they present far lower values than the latter. As discussed in [7], these fluctuations do not belong to a normal turbulent regime. These are then the laminar kinetic energy fluctuations that

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http://dx.doi.org/10.1016/j.amc.2015.07.080
0096-3003/

appear in the upper medium regions of the boundary layer. These streamwise fluctuations are Klebanoff modes identified by Klebanoff in [12]. The production of these fluctuations has particular characteristics. The boundary layer selectiveness of free-stream turbulent eddy scales, filters the broad spectrum free-stream turbulence. This has been identified as "shear-sheltering" and was first described in [13]. The work of Jacobs and Durbin demonstrates this effect using both the discrete and continuous modes of the Orr–Sommerfeld equation, [14]. The continuous modes are eliminated from the main boundary layer region, being confined to the upper reaches of the latter. It was also concluded that the boundary layer penetration depth by the continuous modes is inversely proportional to their frequency. This enforces the concept that only low frequency disturbances are amplified by shear in the pre-transitional laminar boundary layer region. In the experimental work of Volino and Simon, [15], spectra of fluctuating streamwise velocity u', wall-normal velocity v' and turbulent shear stresses $-u'v'$ were recorded. It was found in that work and also in [16], that noticeable values of $-u'v'$ are observed in the pre-transitional boundary layer. These were correlated with peak values of low-frequency wall-normal velocity fluctuations v' of the free-stream turbulence. As previously mentioned these $-u'v'$ values had lower energy and frequency than those found in fully turbulent boundary layer flow. As discussed in the work of Leib et al., [17], Klebanoff first named these streamwise velocity fluctuations as "breathing modes", [12]. In [18], Taylor noticed that these oscillations were related with thickening and thinning of the boundary layer. The production of these streamwise velocity fluctuations u', are believed to be related to the wall-normal velocity oscillations v' through the "splat-mechanism" mentioned by Bradshaw in [19] or by the concept of "inactive motion" proposed by Townsend, [20] and Bradshaw, [21]. As explained by Volino, [22], a negative v' velocity fluctuation imposed by a turbulent eddy will momentarily compress the boundary layer, shifting higher speed flow against the wall surface. This results in an increment of u'. As the turbulent eddy is convected by the flow, the imposed compression effect is diminished resulting in a recovery of the boundary layer to its previous state.

The proposed transition model presents some behaviors of the just described processes. The most noticeable is the prediction of low negative values of $\overline{u'v'}$ in the upper regions of the pre-transitional boundary layer as will be later shown. Also the model predicts when these off-diagonal isotropic Reynolds stress components pierce the laminar-boundary layer. This is then known as the transition onset.

The main purpose of this work is to present the rational behind the development of a new transition model, henceforward designated as V-model. The V-model for transition is then coupled to the Spalart–Allmaras turbulence model and is designated throughout the present work as V-SA. First, the general V-SA model coupling is disclosed. Based on the highlighted physics of transition we then proposed the new mechanical model equations. The description of the used mechanical model approximation is done by first considering some pre-transitional turbulent kinetic energy relations. Afterwards the mechanical model approximation is presented in detail. The development of the transport equation for the pre-transitional turbulent kinetic energy of the proposed transition model is presented. Finally the detailed transition V-model and Spalart–Allmaras turbulence model coupling is described. Initially the model is validated for zero-pressure-gradient bypass transition using the flat-plate T3A and T3B test cases. The latter also includes a validation of the predicted distribution of pre-transitional $\overline{u'v'}$ values for the T3A test case. Then the model is validated for pressure-gradient bypass transition using the flat-plate T3C1, T3C2, T3C3, T3C4 and T3C5 test cases.

2. Mechanical model approximation rational

The rational behind the development of the transition V-model is herein presented, including the flow physics on which it is supported. Before the mechanical approximation disclosure, some considerations need to be taken into account and explained. It is here assumed that pre-transitional turbulence is isotropic in a strain-less free-stream, in the sense that $k_x = k_y = k_z = k_p$. This is in agreement with the flow physics of transition such as presented in the work of [7]. However, under the effect of flow shear the model will predict small pre-transitional negative values of $\overline{u'v'}$ related to non-isotropic turbulence conditions. A bi-dimensional analysis is here described. Nevertheless, the V-model transition closure is applicable to three-dimensional cases. This is the case since the main three orthogonal shear deformation planes are all accounted for in the computation of the local shear magnitude. As such, a global effect of three-dimensionality is taken into account in the computation of the pre-transitional off-diagonal isotropic Reynolds stress components.

2.1. Transition model coupling

The proposed transition V-model focuses on the pre-transitional region depicted in Fig. 1. The developed transition V-model is not able to compute turbulence. Instead it determines the transition onset region depicted in Fig. 1. For this reason and as previously mentioned, the V-model transition closure was coupled to the Spalart–Allmaras turbulence model. Transition onset prediction is performed by computing the viscosity induced by the predicted pre-transitional $\overline{u'v'}$ values described throughout this work. The modus-operandi of the V-SA model is depicted in Fig. 2.

2.2. Pre-transitional turbulent kinetic energy considerations

Pre-transition velocity fluctuations wave forms, for a specific frequency, have seldom a regular shape. Although this is true for most cases, the modeling of the pre-transition region, see works such as [14], requires both discrete and continuum modes. Citing [14], "The eigensolutions to the Orr–Sommerfeld equation in an unbounded domain are classified into two spectra: the first is a finite set of discrete modes; the second is an infinite continuum of modes. The latter are weakly damped and are irrelevant to