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The effect of noise and average relatedness between players in iterated games

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ABSTRACT

In the real world, repetitive game theory has an influential and effective role, especially in political, economic, biological, social sciences and many other sciences. In this work we are exposed to study the effect of noise on the degree of relatedness between the players with respect to the behavior of strategies and its payoff. Our model in this work is the infinitely repeated prisoner's dilemma (PD) game. Because our game is infinitely repeated, we consider any strategy of the game represented by a finite states of automaton (two states). By considering the possibility of a small error in implementation of an automaton, we obtained the payoff matrix for all strategies. Consequently we could identify the behavior of some of the strategies.

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1. Introduction

The theory of repeated games has been an important tool in the behavioral and biological sciences. It provides the central model for explaining how players with selfish objectives might nevertheless be cooperative in the long-term relationship. For that reason, it has been often invoked by economists, anthropologists, political scientists and other scientists who were interested in human cooperation (Axelrod [16]; Aumann [15]; Fudenberg and Maskin [6,7]). The most famous example of repeated games is prisoner's dilemma, and there are also many other iterated games in economics, biology and psychology (E El-Seidy and Kanaya [9]; Rapoport and Chammah [2]; Nowak et al. [14]; Banks and Sundaram [12]).

It is known that natural systems are undeniably subject to random fluctuations, arising from either environmental variability or internal effects. Sun et al. [10]; Li and Jin [13] studied these fluctuations on predator–prey model. They show that the spatially extended system exhibits rich dynamic behavior and that a stationary pattern can be induced to be a stable target wave when the noise intensity is small. Also they show that noise plays a tremendous role in the spread of the disease state, which has implications for how we try to prevent, and eventually eradicate, disease (Sun et al. [11]).

In this paper, we study the repeated prisoner's dilemma game in which there is a relationship between the players, the average relatedness between the players is given by r, which is a number between 0 and 1. A simple way to study games between relatives was proposed by Maynard Smith for the Hawk–Dove game (Hines and Smith [17]; Grafen [1]).

Some of researchers were interested in studying the aspiring to the fittest and promotion of cooperation in the prisoner's dilemma game. Wang and Perc [18] showed that for positive w (noise parameter) players with high payoffs will be considered more likely, while for negative w the opposite holds. Setting w equal to zero returns the frequently adopted random selection of the opponent.

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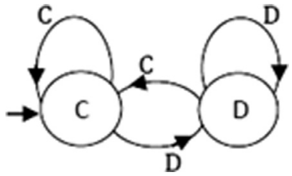

Fig. 1. Tit for tat automata.

In repeated prisoner's dilemma, two players have two options, either to cooperate (C) or to defect (D). In one-shot prisoner's dilemma game, the strategy D is the best, and it dominates the cooperative option. But if this game played repeatedly many times than the picture will change. In this situation the strategy D will not be the dominant strategy for long time. In repeated games, the number of possible grows exponentially with the number of rounds in the game (Nowak et al. [14]; Rubinstein [3,5]). We consider all strategies that can be implemented by deterministic finite state automata with one or two states. The automata define how a player behaves in response to the last move of the other player. We will follow this approach and furthermore, the option of interaction in the real life cannot be implemented without error, because the other player does not necessarily know whether the given action is an error or deliberate option, and a simple error can lead to significant complication. We assume that the automata are subjected to some small error. This error due to implementation or to perception of what the other player does (Zagorsky et al. [4]; Nowak et al. [14]).

In Nowak et al. [14] they studied the prisoner's dilemma where they used the played repeatedly by two-state automata, they computed the 16×16 payoff matrix for limiting case of vanishingly small noise term affecting the interaction.

In this paper, we define the transition rule of each automaton that depends on the initial state of the game and on the payoff of last move, and we compute the payoff matrix of any repeated prisoner's dilemma games in which there is a relationship between the opponents. Then we describe the method that we shall follow to compute the 16×16—payoff matrix for repeated prisoner's dilemma game with noise played by finite state automata.

2. Transition rules

Finite-state automata have been used extensively to study repeated games including the prisoner's dilemma (Zagorsky et al. [4]). In this case, each state of the automaton is labeled by C or D, in the state C the player will cooperate in the next move; in the state D the player will defect. All the strategies start in one of those two states.

Each state has two outgoing transition: one transition specifies what happens if the opponent has cooperated and one if the opponent defected. There are 32 automates with two different initial states, but some of these automata describe automata with the same behavior. Thus there are only 26 automata encoding unique strategies (Nowak et al. [14]; El Seidy et al. [8]).

Each round leads to one of the four possible outcomes (C,C), (C,D), (D,C) or (D,D), where the first position denotes the option chosen by the player and the second that of the co-player. These outcomes, from the player's point of view, are specified by his payoff R, S, T or P, which can be numbered by 1, 2, 3, 4.

The 16 possible transition rules can be defined by a quadruple (p_1, p_2, p_3, p_4) of zeros and ones. Where p_i denotes the probability to cooperate after outcome i ; $i = 1, 2, 3, 4$. Thus (1, 1, 1, 1) is the rule of cooperate in each round (AllC), (0, 0, 0, 0) is the rule of defect in each round (AllD) and (1, 0, 0, 1) for win-stay loss-shift strategy (WSLS), while (1, 0, 1, 0) is the rule of imitating the adversary's last move. The automata using this rule with initial state C (see Fig. 1) is defined by tit-for-tat strategy (TFT). This strategy is very successful in an error-free environment, however in a noisy environment (TFT) achieves a very low payoff against itself since it can only recover from a single error by another error (Zagorsky et al. [4]).

3. Iterated prisoners dilemma between relatives

A simple way to study games between relatives was proposed by Maynard Smith for the Hawk–Dove game (Hines and Smith [17]; Grafen [1]). There are two possible methods to study the games between relatives. The personal fitness method proposed by Grafen [1] modifies the fitness of a player by allowing that a player more likely than the other players of the population to meet a co-player adopting the same strategy as himself. The "inclusive fitness" method adds to the payoff of player r times the payoff of his co-player. We regard the inclusive fitness method to study the repeated prisoner's dilemma that is played by finite state automata and this is subjected to some small error (Hines and Smith [17]).

Consider a population where the average relatedness between players is given by r, which is a number between 0 and 1. In the prisoner's dilemma, two players have two choices, to cooperate (C) or to defect (D). After choosing strategies, the payoff of each player is decided by the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

The meaning of every capitalized letter is follows: R—reward for mutual cooperation, T—temptation to be defected, P—punishment and S—sucker payoff. The condition $T > R > P > S$ is essential, the additional condition $2R > T + S$ should be satisfied.