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New identities and Parseval type relations for the generalized integral transforms $\mathcal{L}_{4n}$,$\mathcal{P}_{4n}$,$\mathcal{F}_{s,2n}$ and $\mathcal{F}_{c,2n}$

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ABSTRACT

In the present paper, the authors consider several new integral transforms including the $\mathcal{L}_{4n}$-transform, the $\mathcal{P}_{4n}$-transform, the $\mathcal{F}_{s,2n}$-transform and the $\mathcal{F}_{c,2n}$-transform as generalizations of the classical Laplace transform, the classical Stieltjes transform, the classical Fourier sine transform and the classical Fourier cosine transform, respectively. Identities involving these transforms are given. Using this identities, a number of new Parseval–Goldstein type identities are obtained. Some examples are also given as illustrations of the results presented here.

1. Introduction, definitions and preliminaries

The Widder potential transform was introduced by Widder [10]. The Widder transform defined the following integral equation:

$$\mathcal{P}\{f(x);y\}=\int_0^\infty \frac{xf(x)}{x^2+y^2}dx. \tag{1}$$

The Widder potential transform is related to the Poisson integral representation of a function which is harmonic in a half plane. The following Laplace type transform which is the $\mathcal{L}_2$-transform,

$$\mathcal{L}_2\{f(x);y\}=\int_0^\infty x\exp\left(-x^2y^2\right)f(x)dx, \tag{2}$$

was introduced by Yürekli [11]. Yürekli and Sadek presented its systematic account in [12]. The $\mathcal{L}_2$-transform and the Laplace transform are related by the formula,

$$\mathcal{L}_2\{f(x);y\}=\frac{1}{2}\mathcal{L}\{f(x^{1/2});y^2\}. \tag{3}$$

The Parseval–Goldstein type formula of the Widder potential transform was given by Srivastava and Singh [8] as follows:

$$\int_0^\infty x\mathcal{P}\{f(u);x\}g(x)dx=\int_0^\infty xf(x)\mathcal{P}\{g(u);x\}dx. \tag{4}$$

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The Parseval–Goldstein type theorem involving the Widder potential transform, the classical Laplace transform and Fourier sine transform were established by Srivastava and Yürekli [9] as follows:

$$\int_0^\infty \mathcal{L}\{f(u);x\}\mathcal{F}_s\{g(u);x\}dx = \int_0^\infty f(x)\mathcal{P}\{g(u);x\}dx. \tag{5}$$

There are numerous analogous results in the literature on integral transforms (see, for example, [3,4,9,11,13]). Dernek et al. [3,4] presented the $\mathcal{L}_4$-transform, the $\mathcal{P}_4$-transform and the $\mathcal{E}_{4,1}$-transform respectively as follows:

$$\mathcal{L}_4\{f(x);y\} = \int_0^\infty x^3 \exp{(-x^4y^4)}f(x)dx, \tag{6}$$

$$\mathcal{P}_4\{f(x);y\} = \int_0^\infty \frac{x^3 f(x)}{x^4+y^4}dx, \tag{7}$$

$$\mathcal{E}_{4,1}\{f(x);y\} = \int_0^\infty x^3 \exp{(x^4y^4)}E_1(x^4y^4)f(x)dx, \tag{8}$$

where $E_1(x)$ is the exponential integral function defined by

$$E_1(x) = -E_i(-x) = \int_x^\infty \frac{\exp{(-u)}}{u}du = \int_1^\infty \frac{\exp{(-xt)}}{t}dt. \tag{9}$$

The $\mathcal{L}_4$-transform is related to the Laplace transform and the $\mathcal{L}_2$-transform by means of the following identities:

$$\mathcal{L}_4\{f(x);y\} = \frac{1}{4}\mathcal{L}\{f(x^{1/4});y^4\}, \tag{10}$$

$$\mathcal{L}_4\{f(x);y\} = \frac{1}{2}\mathcal{L}_2\{f(x^{1/2});y^2\}. \tag{11}$$

The $\mathcal{P}_4$-transform is related to the Stieltjes transform and the Widder potential transform (1) by means of the following identities,

$$\mathcal{P}_4\{f(x);y\} = \frac{1}{4}\mathcal{S}\{f(x^{1/4});y^4\}, \tag{12}$$

$$\mathcal{P}_4\{f(x);y\} = \frac{1}{2}\mathcal{P}\{f(x^{1/2});y^2\}, \tag{13}$$

where the Stieltjes transform is defined as

$$\mathcal{S}\{f(x);y\} = \int_0^\infty \frac{f(x)}{x+y}dx. \tag{14}$$

The $\mathcal{E}_{2,1}$-transform is defined in [1] as

$$\mathcal{E}_{2,1}\{f(x);y\} = \int_0^\infty x \exp{(x^2y^2)}E_1(x^2y^2)f(x)dx. \tag{15}$$

The $\mathcal{E}_{4,1}$-transform (8) is related to the $\mathcal{E}_{2,1}$-transform, the $\mathcal{P}_4$-transform (7) and the $\mathcal{L}_4$-transform (6) by means of the following identities:

$$\mathcal{E}_{4,1}\{f(x);y\} = \frac{1}{2}\mathcal{E}_{2,1}\{f(x^{1/2});y^2\}, \tag{16}$$

$$\mathcal{E}_{4,1}\{f(x);y\} = 4\mathcal{P}_4\{\mathcal{L}_4\{f(x);u\};y\}. \tag{17}$$

The Fourier sine transform and the $\mathcal{F}_{s,2}$-transform are defined respectively as,

$$\mathcal{F}_s\{f(x);y\} = \int_0^\infty \sin{(xy)}f(x)dx, \tag{18}$$

$$\mathcal{F}_{s,2}\{f(x);y\} = \int_0^\infty x\sin{(x^2y^2)}f(x)dx. \tag{19}$$

Dernek et al. [3,4] gave various Parseval–Goldstein type relations between the $\mathcal{L}_2$-transform (2) and the $\mathcal{L}_4$-transform (6) and the $\mathcal{P}_4$-transform (7) and the $\mathcal{F}_{s,2}$ -transform (19) in their articles. The $\mathcal{L}_{2n}$-transform is defined in [5] as follows:

$$\mathcal{L}_{2n}\{f(x);y\} = \int_0^\infty x^{2n-1} \exp{(-x^{2n}y^{2n})}f(x)dx. \tag{20}$$

In this paper, we introduce new generalized integral transforms and establish identities for these integral transforms which are the $\mathcal{P}_{4n}$- transform, the $\mathcal{L}_{4n}$-transform, the $\mathcal{E}_{4n,1}$ -transform and the $\mathcal{F}_{s,2n}$-transform. We define the $\mathcal{P}_{4n}$-transform as

$$\mathcal{P}_{4n}\{f(x);y\} = \int_0^\infty \frac{x^{4n-1}f(x)}{x^{4n}+y^{4n}}dx, \quad n \in \mathbb{N} \tag{21}$$