Applied Mathematics and Computation 269 (2015) 569–583

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Image zooming method using edge-directed moving least squares interpolation based on exponential polynomials

Yeon Ju Lee [a], Jungho Yoon [b,*]

[a] Department of Mathematics, Korea University, Sejong 339-700, South Korea
[b] Department of Mathematics, Ewha Womans University, Seoul 120-750, South Korea

ARTICLE INFO

Keywords:
Moving least squares
Exponential polynomial
Reproducing property
Edge-directed interpolation
Image upsampling
Minimization problem

ABSTRACT

This paper presents a nonlinear image interpolation algorithm. The suggested method is based on the moving least squares (MLS) projection technique, but introduces a fundamental modification. The algebraic polynomial-based MLS methods provide very satisfactory results. However, the associated approximation space is shift-and-scale invariant so that it cannot be adjusted according to the characteristic of a given data. As a result, when upsampling images, it has a limitation in producing sharp edges such that edges are often blurred in the magnified images. To recover sharper edges, we need to reduce smoothing parameter or adapt a new parameter sharpening the edges. Motivated by this observations, we propose a novel MLS method governed by a set of exponential polynomials with tension parameters such that they can be tuned to the characteristic of given data. Moreover, for a better match to the local structures around the edges, the suggested algorithm uses weights which consider the edge orientation. Numerical results are presented and compared, visually and by using some quantitative fidelity measures (PSNR, EPSNR, SSIM and FSIM), to the bicubic spline interpolation and other recently developed nonlinear methods. The results demonstrate the new algorithm's ability to magnify an image while preserving edge features.

1. Introduction

Digital-image interpolation is a fundamental tool in many image processing and biomedical applications such as geometric transformation, demosaicing, registration, satellite-image zooming and texture mapping in computer graphics. Image interpolation methods that are usually employed in image magnification generate missing image pixels at high-resolution from the given low resolution image information. In order for an interpolation scheme to produce a perceivable high quality result in the magnified image, it should preserve the edges, while sharpening them wherever possible, without making jagged edges or ringing artifacts near the boundaries. Moreover, texture regions in the upsampled images should not be blurred or over-smoothened.

Many different image magnification methods have been designed with a broad quality range ([1–3,8,12,16–34]). These methods are categorized as either linear or nonlinear. The linear techniques, such as bilinear and bicubic spline interpolation [1–3] have advantages in simplicity and fast computation, but often suffer from blurring edges or introducing jaggies and ringing artifacts around high frequency feature areas and edges.

Non-linear algorithms have been proposed to improve the subjective quality of the magnified images. Wavelet-based image interpolation methods in [32] and [33] use the (Hölder) regularities of edges which are measured from the decay of wavelet

* Corresponding Author. Tel.: +82 2 3277 2293; fax: +82 2 3277 2289.
E-mail addresses: leeyeonju08@korea.ac.kr (Y.J. Lee), rbf003@gmail.com, yoon@ewha.ac.kr (J. Yoon).

http://dx.doi.org/10.1016/j.amc.2015.07.086
0096-3003/

transform coefficients across multi-scales. These works were extended by using the entire cone influence of a sharp edge in wavelet scale space to estimate the finest scale coefficients through optimal recovery theory [34]. The technique in [19] determines the local quadratic signal from the local patches, then estimates the missing pixels by applying optimal recovery. In [18], a sub-pixel edge estimation is exploited to generate a high resolution (HR) edge map from the low resolution (LR) image, and then the high resolution edge map is used to correct the interpolated pixel. A nonlinear method using rational filter was also proposed in [26] and the concept of warped distances was introduced to adaptively adjust the bicubic interpolation method [27]. The algorithm in [28] uses the local covariance of the LR image to estimate the covariance of a HR image, and then the estimated covariance is exploited to adapt the interpolation at a HR based on the geometric duality between the LR and the HR covariances.

The method in [29] interpolates a missing sample in two orthogonal directions, and then fuses the directional interpolation results by the linear minimum mean square-error estimation. A 2-D piecewise autoregressive model is used to model the image structure and reconstructs the image via soft-decision estimation [30]. This technique is equivalent to interpolation using an adaptive nonseparable 2-D filter of high order. The work of [14] improves the robustness of soft-decision adaptive interpolation by using weighted least-squares estimation instead of least-squares estimation in (parameter and data) estimation steps. The authors of [31] proposed the error amended sharp edge (EASE) scheme based on a modified bilinear interpolation, where the interpolation error is corrected by using the *interpolation error theorem* in an edge-adaptive fashion. The moving least squares (or known as kernel regression) methods in polynomial space have been proved quite useful in image interpolation [8,12]. The authors of [13] proposed a regularized local linear regression algorithm, and the samples are weighted by combining nonlocal means and bilateral filter. Instead of algebraic polynomials, the Gaussian radial basis function is employed for an edge-directed image upsampling method [17]. The interpolation in [16] is implemented by using compactly-supported exponential splines, and the properly-tuned signal-dependent kernels perform better than polynomial B-spline models. Moreover, an edge-directed interpolation method is formulated as a PDE-based problem [20,23–25] and as a total-variation regularization [21,22]. Although these non-linear interpolation methods can produce clearly visible edges as compared to those produced by liner methods, most of these methods are computationally expensive (e.g., [28]) and suffers from the limitation of angle resolution (e.g., [17]).

The main objective of this study is to provide a novel non-linear moving least squares (MLS) techniques for image upsampling with some fundamental modifications. Although the known MLS methods provide very satisfactory results, the associated approximation space is shift-and-scale invariant so that it cannot be adjusted according to the characteristic of a given data. Thus, when upsampling images, it has a limitation in producing sharp edges; some blurring effects are introduced in the magnified images. In order to recover sharper edges with these methods, we need to reduce smoothing parameter or adapt a new parameter sharpening the edges. Motivated by this observations, we propose a novel MLS method based on a set of exponential polynomials which have tension parameters such that they can be tuned to the characteristic of given data, yielding better upsampling results than the other known methods. Moreover, for a better match to the local structures around the edges, the suggested algorithm uses weights which consider the edge orientation. The proposed algorithm is as simple to implement as linear methods but it demonstrates improved visual qualities by preserving the edge features. It can be applied to image magnification in arbitrary (integer and non-integer) resolution zooming factors. Last but not least, although this paper is concerned with uniformly sampled images, the suggested method can be extended to treat images with irregular samples (e.g., geometrically transformed images).

The following notations are used in this paper. Let $\mathbb{Z}_+^2 := \{\mathbf{s} := (s_1, s_2) \in \mathbb{Z}^2 \,:\, s_1, s_2 \geq 0\}$. For $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$ and $\mathbf{s} \in \mathbb{Z}_+^2$, we set $\mathbf{x}^{\mathbf{s}} = x_1^{s_1} x_2^{s_2}$, and $|\mathbf{s}|_1 := s_1 + s_2$. The space of algebraic polynomials of degree m in $\mathbb{R}^2$ is denoted by Π_m. For a given set S, $\#S$ indicates the number of elements of a set S.

The rest of this paper is organized as follows. Section 2 presents a brief introduction of the MLS method and the motivation to develop a new MLS method. A novel non-linear MLS scheme based on exponential polynomials is presented in Section 3. Section 4 provides some numerical experiments by comparing the proposed method with other (linear and nonlinear) upsampling methods.

2. General overview: the classical MLS

Suppose that our observed image is a discrete sampling of a function $f : \mathcal{D} \to \mathbb{R}$ at an equally spaced point set $X := \{\mathbf{x}_n : n = 1, \ldots, J\}$ in a rectangle $\mathcal{D} \subset \mathbb{R}^2$. A powerful tool to approximate an unknown function from a sample set is to fit local polynomials by the moving least-squares methods [5,8]. In fact, during the last decades, some approaches similar to the MLS techniques have been studied in the different names such as *bilateral filter* [10,11], *normalized convolution* [9], and *kernel regression* [12].

In the classical MLS method, the optimal fitting is expressed as a linear combination of polynomials. For a point $\mathbf{x} \in \mathcal{D}$, the approximation values $p^*(\mathbf{x})$ (i.e., the coefficients of the basis function) are obtained by solving the following quadratic minimization problem:

$$p^*(\mathbf{x}) := \underset{p \in \Pi_m}{\mathrm{argmin}} \left\{ \sum_{n=1}^{J} \| p(\mathbf{x}_n) - f(\mathbf{x}_n) \|^2 w(\|\mathbf{x} - \mathbf{x}_n\|) \right\}, \tag{1}$$

where w is a radial and fast decreasing smooth function and $J > \dim \Pi_m$ and $\|\cdot\|$ is the Euclidean distance in $\mathbb{R}^2$.

The MLS methods in polynomial space have been proved to be quite useful in image interpolation as well as denoising and superresolution [8,12]. However, as is evident, the classical method gives similar penalties to data within a similar distance from the evaluation point. As such, neighboring data around the evaluation point are heavily weighted even though the data at these points does not necessarily have any similarity. As a result, when upsampling images across edges, some artifacts (blurring