Applied Mathematics and Computation 269 (2015) 904–929

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Particle swarm optimization with stochastic selection of perturbation-based chaotic updating system

Keiji Tatsumi*, Takeru Ibuki, Tetsuzo Tanino

Graduate School of Engineering, Osaka University, Osaka, Japan

A R T I C L E I N F O

Keywords:
Chaotic system
Particle swarm optimization
Metaheuristics
Snap-back repeller
Global optimization

A B S T R A C T

In this paper, we consider the particle swarm optimization (PSO). In particular, we focus on an improved PSO called the CPSO-VQO, which uses a perturbation-based chaotic system and a threshold-based method of selecting from the standard and chaotic updating systems for each particle on the basis of the difference vector between its pbest and the gbest. Although it was reported that the CPSO-VQO performs well, it is not easy to select an amplitude of the perturbation and a threshold appropriately for an effective search. This is because the bifurcation structure of the chaotic system depends on the difference vector, and the difference vector varies widely between different stages of the search and between different problems.

Therefore, we improve the CPSO-VQO by proposing a modified chaotic system whose bifurcation structure is irrelevant to the difference vector, and show theoretically desirable properties of the modified system. We also propose a new stochastic method that selects the updating system according to the ratio between the components of the difference vector for each particle, and restarting and acceleration techniques to develop the standard updating system used in the proposed PSO model. The proposed methods can maintain an appropriate balance between the identification and diversification aspects of the search. Moreover, we perform numerical experiments to evaluate the performance of the proposed PSOs: PSO-TPC, PSO-SPC, PSO-SDPC, IPSO-SPC and IPSO-SDPC. In particular, we demonstrate that the IPSO-SDPC finds high-quality solutions and is robust against variations in its parameter values.

1. Introduction

Particle swarm optimization (PSO) is a metaheuristic method for global optimization which is inspired by the behavior of a swarm of birds or fish [7]. The PSO searches for solutions by simultaneously updating a number of candidate solutions called *particles*. This method is very simple and performs well at finding desirable solutions, but it is known to sometimes prematurely converge to an undesirable local minimum. In order to improve the diversity of the search in the PSO, various kinds of models have been investigated [4,19]. In this paper, we focus on variants of the PSO which exploit a chaotic system in order to improve their ability to explore. Many of these methods use chaotic sequences to update the positions of the particles, in which particles search for solutions extensively because of the chaoticity. It has been reported that these PSO variants have a more diverse search than does the standard PSO [1,3,10,17].

* Corresponding author. Tel.: +81 6 6879 7787; fax: +81 6 6879 7939.
E-mail address: tatsumi@eei.eng.osaka-u.ac.jp (K. Tatsumi).

http://dx.doi.org/10.1016/j.amc.2015.07.098
0096-3003/

In this paper, we focus on the *chaotic PSO exploiting a virtual quartic objective function based on the personal and global best solutions*(CPSO-VQO)[17]. This model uses a perturbation-based chaotic system that is derived from a quartic tentative objective function by using the steepest descent method with a perturbation. The function is determined for each particle, and it has two global minima at the pbest of the particle and the gbest. In CPSO-VQO, each component of the position of the particle is updated by either the chaotic system or the updating system of the standard PSO; if the absolute value of a component of the difference vector between the gbest and pbest of the particle is greater than a threshold, then the corresponding component of the position is updated by the chaotic system; otherwise, it is updated by the standard one. It was reported that CPSO-VQO is able to maintain a diverse search due to the chaotic nature of the search, and thus it performs better than do the other PSOs. Moreover, in [17], Tatsumi et al. presented a sufficient condition for chaoticity of the system used in CPSO-VQO and showed that the parameter values in the system can be selected by utilizing its bifurcation diagram. In particular, the numerical experiments showed that it is important to select an appropriate amplitude of the perturbation so that the chaotic system generates a sequence which moves around the gbest and pbest without being trapped at undesirable local minima. Such a behavior arises when the chaotic system has a strange attractor that includes the gbest and pbest.

However, CPSO-VQO has some drawbacks. In this paper, we first point out them. In CPSO-VQO, it is not easy to select an appropriate amplitude because the bifurcation structure of the system depends heavily on the difference vector between the gbest and pbest, and the difference vector varies widely between different particles, between different stages of the search and between different problems. In addition, it is important to balance the intensification and the diversification of the search, and in CPSO-VQO, the balance is determined by the chaotic updating rate, which indicates the rate of the total number of components of positions of particles updated by the chaotic systems, to the total number of components of positions of all the particles. However, the only way for adjusting the rate is through the selection of the threshold value, which, as mentioned, is difficult to choose due to the variations in the difference vectors.

Therefore, we propose a chaotic system with a modified perturbation term whose amplitude and angular frequency are proportional and inversely proportional, respectively, to the corresponding component of the difference vector. We then show theoretically the sufficient conditions for the modified system to be chaotic. Moreover, we verify, both theoretically and experimentally, that the bifurcation structure of the modified system does not depend on the difference vector between the gbest and pbest; this property makes it easier to select suitable parameter values in the system for global optimization. We also propose a new PSO called the *PSO with threshold-based selection for perturbation-based chaotic updating system* (PSO-TPC), which uses the modified chaotic system and a threshold-based selection method that is similar to that of CPSO-VQO. Next, we propose a new method for the stochastic selection of the updating system; in this method, the updating system for each component of the position of each particle is selected according to the ratio of the absolute value of the corresponding component to the maximal absolute value in all components of the difference vector. In this model, the chaotic updating rate can be more explicitly controlled, which is expected to make it easier to keep an appropriate balance between intensification and diversification. The PSO with the modified chaotic system and the proposed selection method is called the *PSO with stochastic selection for perturbation-based chaotic updating system* (PSO-SPC). Furthermore, we add a technique to the PSO-SPC that uses a high chaotic updating rate during the early stages of the search, and then decreases it exponentially. This model, which is expected to maintain a better balance between intensification and diversification than the PSO-SPC, is called the *PSO with stochastic selection and decreasing chaotic updating rate for perturbation-based chaotic updating system* (PSO-SDPC). In addition, we develop the standard updating system used in the proposed PSO by introducing techniques of reinitializing the particle's velocity and accelerating the convergence of the search. The proposed PSOs, PSO-SPC and PSO-SDPC, using the developed updating system are called IPSO-SPC and IPSO-SDPC. Finally, through numerical experiments, we evaluate the performance of the proposed PSOs and compare them with the existing PSOs, and, in particular, we show that the IPSO-SDPC has an advantage in finding desirable solutions for various large-scale benchmark problems.

This paper is organized as follows: in Section 2, we introduce the standard PSO, the improved PSOs, and the CPSO-VQO. In Section 3, we modify the perturbation term of the chaotic system and show theoretically its desirable properties, and we also propose new stochastic selection methods, acceleration and reinitialization techniques. In Section 4, we use numerical experiments to verify the performance of the five proposed PSOs for some benchmark problems. Finally, we present our conclusions in Section 5.

2. Chaotic particle swarm optimization

2.1. Particle swarm optimization

In this paper, we focus on the following global optimization problem, which has many local minima and a rectangular constraint:

$$\text{(P)} \quad \min \; f(x) \quad \text{s.t. } x \in X := \prod_{i=1}^{n} [x_i^l, x_i^u].$$

In order to solve this problem with PSO, a number of candidate solutions, called *particles*, are simultaneously updated by exchanging information with each other. At each iteration t, particle i moves toward a linear combination of two tentative solutions, *pbest* $p^i(t)$ and *gbest* $g(t)$, where the former is the best solution obtained by particle i until iteration t and the latter is the best