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Image processing using Newton-based algorithm of nonnegative matrix factorization ☆

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ABSTRACT

In this paper, we propose a Newton-based algorithm for nonnegative matrix factorization in image processing. We employ the new algorithm to three real-world databases. Extensive numerical results show the feasibility and validity of the proposed algorithm.

1. Introduction

Nonnegative matrix factorization (NMF) [1–6] decomposes the nonnegative data matrix $V \in R^{m\times n}$ as a product of two nonnegative matrices $W \in R^{m\times r}$ and $H \in R^{r\times n}$ such that the cost function

$$f(W, H) := \frac{1}{2}\|V - WH\|_F^2 \tag{1.1}$$

is minimized, where $r \ll \min(m, n)$ is a given positive integer and $\|\cdot\|_F$ represents the Frobenius norm of the corresponding matrix.

NMF plays an important role in many real applications, such as extraction and identification applicable to image processing [1,2,5,6], text mining [7–10], spectral data analysis [11], speech processing [12,13], ail quality analysis, and so on.

In recent years, numerous algorithms have been proposed to solve the NMF problem in Eq. (1.1). Most of these algorithms can be broadly classified into two categories: alternating one-step gradient descent and alternating nonnegative least squares.[14] The former includes the multiplicative update (MU) algorithm proposed by Lee and Seung [1]. This kind of algorithm is to alternately update W and H with one step. This is in spirit similar to the Hermitian and Skew–Hermitian splitting iteration method for solving linear equations [15–18]. Although these gradient descent algorithms are easy to implement, they lack convergence guarantee. The alternating nonnegative least squares is to alternately update W and H by solving the corresponding nonnegative least squares problems. Lin proposed the columnwisely alternating projected gradient(cAPG) algorithm and the elementwisely alternating projected gradient (eAPG) algorithm in [2] for NMF problem in Eq. (1.1). Both cAPG and eAPG are alternating nonnegative least squares algorithms. Newton-type algorithms [19,20] are also belong to the nonnegative least squares algorithms. Kim et al.[20] proposed an algorithm which is named fast nonnegative matrix approximationly–inexact ($FNMA^I$) algorithm. But this algorithm made an assumption that both W and H had full column ranks. Thus W^TW and HH^T are positive definite. This is likely to happen when rank r is very small. If rank r is not very small or W and H are relatively sparse, W^TW and HH^T may be irreversible. In view of this situation, this paper propose an algorithm which combine cAPG algorithm with $FNMA^I$ algorithm in

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image processing and employ it to three real-world databases: MIT Center for Biological and Computation Learning Face Database (CBCL) [21], Olivetti Research Laboratory Face Database (ORL) [22] and Center for Biometrics and Security Research Iris Database (IRIS) [23].

The rest of this paper is organized as follows. In Section 2, we review MU algorithm proposed by Lee and Seung [1], and cAPG algorithm, which was proposed by Lin [2]. The proposed algorithm, a Newton-based algorithm for nonnegative matrix factorization is detailed in Section 3. Some numerical results, compared with the existing two algorithms proposed by Lin and Liu [5], are presented in Section 4. The paper is finally concluded in Section 5.

2. Preliminaries

The classical algorithm for NMF is the following multiplicative update algorithm (MU) proposed by Lee and Seung [1].

Algorithm 2.1 (The MU Algorithm)**.**

Step 1 *Set* $W := rand(m, r)$ *and* $H := rand(r, n)$;
Step 2 *For iter=1:maxiter, compute alternately*

$$H := H. * (W^T V)./(W^T WH),$$
$$W := W. * (VH^T)./(WHH^T).$$

Here, the operations .∗ and ./ denote the elementwise multiplication and division, respectively.

As we known, at each iteration step the MU algorithm alternates between the gradients of the cost function $f(W, H)$ with respect to H and W, for a fixed W the NMF problem (1.1) is a constrained least-squares problem with respect to H.

The following is a derivation of the updating formula about H involved in the MU algorithm.

$$H_{ij} \leftarrow H_{ij} - \eta_{ij}[\nabla_H f(W, H)]_{ij}, \tag{2.1}$$

where

$$\nabla_H f(W, H) = W^T(WH - V),$$

and $[\nabla_H f(W, H)]_{ij}$ denotes the (i, j)th element of the gradient matrix $\nabla_H f(W, H)$. To zero the potentially negative part in (2.1), i.e., $H_{ij} - \eta_{ij}[W^T WH]_{ij}$, we may choose

$$\eta_{ij} = \frac{H_{ij}}{[W^T WH]_{ij}}.$$

Then we have:

$$H_{ij} \leftarrow \frac{H_{ij}[W^T V]_{ij}}{[W^T WH]_{ij}}.$$

Analogously, we have another formula of MU algorithm.

$$W_{ij} \leftarrow \frac{W_{ij}[VH^T]_{ij}}{[WHH^T]_{ij}}.$$

The following is the derivation of the columnwisely alternating projected gradient (cAPG) algorithm in [2].

For a fixed W, Let

$$\begin{aligned} \eta_j^* &= \arg\min_{\eta_j} \frac{1}{2}\|V_j - W(H_j - \eta_j[\triangledown_H f(W, H)]_j)\|^2 \\ &= \frac{\|[\triangledown_H f(W, H)]_j\|^2}{\|W[\triangledown_H f(W, H)]_j\|^2}, \end{aligned} \tag{2.2}$$

where V_j, H_j, $[\triangledown_H f(W, H)]_j$ denote the jth column of the matrices V, H and $\triangledown_H f(W, H)$, respectively. The updating formula of cAPG algorithm with respect to H is

$$H_j \leftarrow H_j - \eta_j^*[\nabla_H f(W, H)]_j.$$

Analogously, we have another formula of cAPG algorithm.

$$W_i^T \leftarrow W_i^T - \xi_i^*[\nabla_W f(W, H)]_i^T,$$

where