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# Comparison of deterministic and probabilistic approaches to identify the dynamic moving load and damages of a reinforced concrete beam

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ABSTRACT

Two classical civil engineering inverse problems are considered. The first deals with the determination of dynamic moving loads applied to a reinforced concrete beam. The second one corresponds to the monitoring and the damage assessment. The concrete damage due to overloading is modeled by a loss of the concrete Young' modulus, whereas the steel bar damage due to corrosion effects is modeled by a reduction of the steel bar cross section. To identify the loading and damage parameters, deterministic and probabilistic model updating techniques are applied and compared. In the deterministic approach, a gradient descent technique based on the adjoint framework is used to minimize the data misfit functional with a Tikhonov regularization term. Then, a regularization by a means of Bayes's rule is considered in a probabilistic approach. The estimation is of the minimum variance type achieved with the help of the transformed ensemble Kalman filter.


## 1. Introduction

Structural Health Monitoring is particularly useful to detect and localize damages, to reduce the maintenance cost of structures and to ensure user safety. To prevent early damages, smart systems have been developed to identify overloaded vehicles on civil engineering structures such as bridges and viaducts. One particular example of these is the Bridge Weigh-In-Motion (B-WIM) system that has been studied for the last 30 years [1–3]. Concerning the damage identification, according to [4] one may distinguish four categories: the detection of damage (level 1), localization (level 2), quantification of the damage (level 3), and lifetime prediction update (level 4). Categories of level 1 and 2 can be achieved with the help of the data driven methods, such as for example vibration-based techniques. For this purpose one can use modal-based [5,6] or static based statistical approaches [7,8]. To accomplish higher level categories, one requires model updating techniques. The parameter (model) estimation from the noisy indirect sensor outputs is not an easy task. As the problem is generally ill-posed, it requires a certain kind of regularization [9]. In the deterministic sense, the regularization is very often achieved by a Tikhonov regularization [10]. However, other possible techniques also exist, for example Constitutive Relation Error (CRE) regularization [11–13] previously studied by one of authors. On the other hand, the ill-posed problem can be regularized in a probabilistic manner via Bayes rule by adding the prior expert knowledge on the parameter (model) set next

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to the observation data. Most of these approaches are based on the Monte Carlo kind of sampling procedures, such as for instance Markov Chain Monte Carlo techniques [14,15]. In recent years, another more simple kind of methods has appeared. This often assumes linearity of the observation operator and Gaussian noise—the linear Bayesian filter—such as the ensemble Kalman filter [16] and its generalization in the form of polynomial chaos based linear filter [17,18]. In case of nonlinearity these can be extended to more complex and accurate forms as presented in [19,20].

The objective of this paper is to qualitatively and not quantitatively compare deterministic [21] and probabilistic [17] updating techniques. For this purpose Tikhonov regularization and ensemble Kalman filter procedures are applied on two important civil engineering applications: the identification of a moving load on a reinforced concrete beam, and the detection of damages in the concrete and steel bar of the beam. The identification is performed by using the full temporal data read by strain sensors and the structural dynamic model based on the partial differential equation.

The article is organized as follows: Section 2 summarizes the motivation behind this work and focuses on the description of two inverse civil engineering problems of huge practical importance. Section 3 lays out the mathematical dimension of the numerical approaches used in this research, and Section 4 ties up the numerical findings of both deterministic and probabilistic computational approaches.

## 2. Model problem

A simplified model of a two-dimensional (2D) concrete beam occupying the domain $\Omega$ with a single perfectly adherent horizontal steel bar $\Gamma$ is considered (see Fig. 1) in the time interval $[0, T]$. The beam is supported on the boundaries $\partial\Omega_{i1}$ and $\partial\Omega_{i2}$ and further assumed to be under plain strain and small perturbation assumption conditions. A dynamic moving load $\underline{F}_d(\chi, t)$ is applied to the top $\partial\Omega_{Fd}$ of the concrete beam, whereas $\Omega_{F0}$ represents the stress-free boundary condition.

The mechanical model is described by the following partial differential equations (PDEs).

- Dynamic equilibrium equations:

$$\rho_c \underline{\ddot{u}} - div(\underline{\underline{\sigma}}) = \underline{0}, \text{ in } \{\Omega - \Gamma\} \times [0, T] \tag{1}$$

$$\rho_b S_b \ddot{u}_x - \frac{\partial N}{\partial x} - [\![\sigma_{xy}]\!]_\Gamma = 0, \text{ in } \Gamma \times [0, T] \tag{2}$$

$$\rho_b S_b \ddot{u}_y - [\![\sigma_{yy}]\!]_\Gamma = 0, \text{ in } \Gamma \times [0, T] \tag{3}$$

- Boundary conditions:

$$\underline{\underline{\sigma}}\ \underline{n}_{|\partial\Omega_{F0}} = \underline{0}, \text{ in } \partial\Omega_{F0} \times [0, T] \tag{4}$$

$$\underline{\underline{\sigma}}\ \underline{n}_{|\partial\Omega_{Fd}} = \underline{F}_d(\chi, t), \text{ in } \partial\Omega_{Fd} \times [0, T] \tag{5}$$

$$\underline{u} = \underline{0}, \text{ in } \partial\Omega_{i1} \times [0, T] \tag{6}$$

$$\underline{u} \cdot \underline{y} = 0, \text{ in } \partial\Omega_{i2} \times [0, T] \tag{7}$$

$$\underline{\underline{\sigma}}\ \underline{n}_{|\partial\Omega_{i2}} \cdot \underline{x} = 0, \text{ in } \partial\Omega_{i2} \times [0, T] \tag{8}$$

- Initial conditions:

$$\underline{u}(t = 0) = \underline{0}, \text{ in } \Omega \tag{9}$$

$$\underline{\dot{u}}(t = 0) = \underline{0}, \text{ in } \Omega \tag{10}$$

- Constitutive relation:

$$\underline{\underline{\sigma}} = \mathcal{K}_c : \underline{\underline{\epsilon}}(\underline{u}), \text{ in } \Omega \times [0, T] \tag{11}$$

$$N = E_b S_b \frac{\partial u_x}{\partial x}, \text{ in } \Gamma \times [0, T] \tag{12}$$

In the following, the system of PDEs (1)–(12) is called "direct problem". In the governing equations, $\rho_c$ (resp. $\rho_b$) represents the volumic mass of the concrete (resp. steel bar), $S_b$ and $E_b$ are the area of the steel bar cross-section and corresponding Young's modulus, $u_x$ and $u_y$ denote the components of the displacement vector $\underline{u}$, $N$ is the tension in the steel bar $\Gamma$, and $\underline{n}_{\partial\Omega}$ is the exterior normal vector to the boundary. Furthermore, $[\![\bullet]\!]_\Gamma$ is the jump of $\bullet$ on $\Gamma$, and $\dot{X} = \partial X/\partial t$ and $\ddot{X} = \partial^2 X/\partial t^2$ are used to denote the partial derivatives of some considered variable $X$ over time. Finally, $\underline{\underline{\sigma}}$ and $\underline{\underline{\epsilon}}$ are symmetric stress and strain second order tensors, respectively. Due to the small strain assumption, the strain matrix can be computed via the displacement vector $\underline{u}$

$$\underline{\underline{\epsilon}}(\underline{u}) = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{yy} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_x}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}\right) & \dfrac{\partial u_y}{\partial y} \end{bmatrix} \tag{13}$$