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An adjoint-based Jacobi-type iterative method for elastic full waveform inversion problem

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ABSTRACT

Full waveform inversion (FWI) is a promising technique that is capable of creating high-resolution subsurface images of the earth from seismic data. However, it is computationally expensive, especially when 3D elastic wave equation is considered. In this paper an adjoint-based computational algorithm has been proposed to address the computational challenges. The important strategy in this work is to decouple the two Lamé parameters so that two half-sized subproblems are resulted and solved separately. Mathematically, the inverse problem is formulated as a PDE-constrained optimization problem in which the objective functional is defined as the misfit between observational and synthetic data. The parameters to be recovered are the spatially varying Lamé parameters which are of great interests to geophysicists for the purpose of hydrocarbon exploration and subsurface imaging. The gradient of the misfit functional with respect to the Lamé parameters is calculated by solving the adjoint elastic wave equation, whilst the Quasi-Newton method (L-BFGS) is used to minimize the misfit functional. Numerical experiments demonstrated that the new method is accurate, efficient and robust in recovering Lamé parameters from seismic data.

1. Introduction

Elastic wave equation is a set of partial differential equations (PDE) that have been widely used to describe the wave propagation in an elastic medium such as the earth. For a heterogeneous isotropic medium, two parameters, λ and μ, the first and second Lamé parameters are of great interests to Geophysicists, Mathematicians and Engineers working in the areas of seismic imaging and oil exploration. For example, the two parameters can be used to determine the P- and S-wave velocities. Despite the great efforts that had been devoted, it still remains a very challenging task to recover those parameters using recorded seismograms, which are usually available on part of the geophysical domain.

FWI has gained much popularity and shown great potential in creating high-resolution subsurface images of the earth from seismic data. The FWI was established by Lailly [16] and Tarantola [32] for acoustic wave equation, then for elastic wave equation [33]. These methods solve the wave equations in time domain. Later on, the FWI was implemented by Pratt in frequency domain [24,25]. A complete introduction of the up-to-date developments of FWI is given by Fichtner [11]. An overview of recent developments of FWI is also given by Virieux and Operto [35]. Some recent research work and developments in elastic full waveform inversion have been reported as well [5,8,27,36]. Despite the great improvements in the past several decades, some technical difficulties and challenges still remain unsolved or just partially resolved. Among these are the extremely high computational cost and huge memory requirement, due to the large size of geophysical domain in real applications and huge amount of seismic data being processed.

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FWI is usually solved through solving a PDE-constrained optimization problem, which is a powerful technique that has been widely used in various applications including geophysical inversion and compute tomography problems [1,3,4]. A typical workflow of FWI is an iterative procedure which consists of several main components: forward modeling, adjoint modeling, gradient calculation and optimization. Unfortunately this procedure is a computationally challenging task which requires huge memory storage and computational time. Hence, it is always desirable to reduce the number of iterations in FWI. To this end, several strategies have been used in the past to reduce the number of iterations, such as choosing an accurate initial model, generating an accurate gradient and utilizing fast optimization algorithms [26].

The central piece of FWI is the optimization procedure which searches for the optimal updates of the parameters. The global optimization algorithms are superior to local optimization methods in terms of avoiding local minima as they attempt to find the global minimum of the misfit functional, hence the search procedure is unlikely to be trapped by local minima [30]. Tran and Hiltunen successfully applied the simulated annealing algorithm in a 2D full waveform inversion for elastic medium [34]. However, the convergence of global optimization methods such as the simulated annealing and the genetic algorithm, is slow and even not guaranteed. Given that FWI is a computationally intensive procedure, local optimization methods such as conjugate-gradient algorithm and Quasi-Newton algorithm are preferred. To avoid explicit computing of inverse Hessian matrix, we adopt an efficient implementation of Quasi-Newton method, the L-BFGS algorithm, in which only a few vectors(gradients in the past iterations) are stored and used to approximate the inverse Hessian matrix for the next iteration [19,21]. This implementation leads to significant saving in memory storage and computing time.

To implement a gradient-based local optimization algorithm(L-BFGS in this case), the gradient of the misfit functional with respect to the Lamé parameters is required. Several approaches are available for efficient gradient computation. One choice is using the popular automatic differentiation (AD) tools such as TAMC [12,13], ADIFOR [2] and TAPENADE [15]. However AD tools are not efficient for large-scale problems involving complicated numerical forward modeling. Another option is the adjoint state method, in which the adjoint equation is derived and numerically solved to calculate the gradient. It has been shown that if all numerical procedures are implemented appropriately, the results generated by AD tools and adjoint state method should be identical up to machine precision. The adjoint state method has a long history in solving constrained optimization problems. Recently it has been widely used in solving various inverse problems arise in science and engineering, especially in geoscience [6,9,14,17,18,31]. A complete review of the recent advances about adjoint state method in geophysical applications can be found in [23]. To effectively resolve the issue on the huge memory requirement, we can implement the checkpointing technique in adjoint wavefields simulation. To validate the gradients generated by the adjoint state method, the automatic differentiation tool TAMC is implemented for comparison on a small-scale model.

In this paper we develop a new computational method to solve the FWI problem by formulating it as an elastic wave equation constrained optimization problem, which is solved by the adjoint state method. The new method features the Jacobi-type iterative procedure in which the two Lamé parameters are optimized alternatively during each iteration, to reduce the memory storage and enable parallel implementation. The rest of the paper is organized as the follows. In Section 2, the FWI problem and the corresponding PDE-constrained optimization problem are introduced, as well as the governing elastic wave equations. In Section 3, the adjoint analysis is applied to derive the adjoint elastic equations and the formula to calculate the gradients with respect to the two Lamé parameter, which is followed by the description of the numerical algorithms in Section 4. Two numerical examples are solved to demonstrate the efficiency and accuracy of the new method in Section 5. Finally some remarks and conclusions are addressed in Section 6.

2. Mathematical formulation

We first formulate the FWI problem as an elastic wave equation constrained optimization problem. Consider the 3D elastic wave equation

$$\rho\frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla\lambda(\nabla\cdot\mathbf{u}) + \nabla\mu\cdot[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda+2\mu)\nabla\nabla\cdot\mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u} + s(x,y,z,t), \quad (x,y,z,t)\in\Omega\times[0,T], \tag{1}$$

$$\mathbf{u}|_{t=0} = \mathbf{u}_0(x,y,z), \quad \mathbf{u}_t|_{t=0} = \mathbf{u}_1(x,y,z), \quad (x,y,z)\in\Omega, \tag{2}$$

$$\mathbf{u}|_{\partial\Omega} = \mathbf{u_b}(x,y,z,t), \quad (x,y,z,t)\in\partial\Omega\times[0,T], \tag{3}$$

where $\mathbf{u} = (u_x, u_y, u_z)$ represent the displacements in x-, y- and z-directions, $\lambda(x,y,z)$ and $\mu(x,y,z)$ are the Lamé parameters, ρ is the density of the medium which is assumed to be a constant in this work. $s(x,y,z,t)$ is the source function that generates the wave. $\mathbf{u_b}$ is a function describing the Dirichlet boundary condition. In a more realistic situation, the free-surface boundary condition or absorbing boundary condition are usually implemented. However it noted that the new method can be adjusted to any other types of boundary conditions. Suppose that the observational data $\bar{\mathbf{u}}$ is available on the boundary $\partial\Omega$ (It is worth mentioning that in real applications the observational data are only available on part of $\partial\Omega$, for instance the surface), the inverse problem is to seek λ and μ in an appropriate function space such that the computed wavefields match the observational data $\bar{\mathbf{u}}$. To this end, we define the following misfit functional

$$\mathcal{J}(\lambda,\mu) = \frac{1}{2}\int_0^T\int_{\partial\Omega}|\mathbf{u}(\lambda,\mu,x,y,z,t) - \bar{\mathbf{u}}|^2\,dSdt + \mathcal{R}(\lambda,\mu), \tag{4}$$