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A mortar element approach on overlapping non-nested grids: Application to eddy current non-destructive testing

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ABSTRACT

The paper presents a finite element approach involving strongly coupled field approximations on moving non-matching overlapping grids. It generalizes a preliminary version of the mortar element method on overlapping grids (Maday et al., 2003) to the case where the field source can be in the moving domain and the physical parameters of the moving domain can differ from those of the surroundings. This generalization has been developed to face the need of some industrial sectors for an efficient investigation numerical method in eddy current non-destructive testing.

1. Introduction

Since the 1950s the technology related to non-destructive testing (NDT) by eddy currents (EC) has developed increasingly. Metal detectors are also used (since the end of the first world war) for security screening and more recently to detect foreign bodies in food. Highly sensitive eddy current sensors and instruments are now widely used especially in the aircraft and nuclear industries for structures' integrity inspection. Eddy current testing is particularly fast at automatically inspecting products. Indeed, inspection can be implemented without any direct physical contact between the sensor and the inspected material. Moreover, inspection results are instantaneous. This technique works mainly with the large variety of non-ferromagnetic or ferromagnetic metals, used in industry, but also with other electrically conductive materials (e.g. carbon fiber reinforced polymers).

EC NDT relies on the interaction between a magnetic field source and a piece of electrically conductive material, without causing any damage to the tested piece. An alternating current feeds a coil (eddy current sensor) constituted of several turns of insulated copper wires with or without a ferromagnetic core, and produces an alternating magnetic field (excitation field). If a conducting metallic piece is close to the coil, eddy currents will be induced in the metal, and this produces a magnetic field of its own (reaction field). The presence of small voids (absence of material) such as cracks in the piece as well as inclusions (presence of foreign materials) hidden within objects or buried underground, can thus be detected, as long as they create a contrast in the electrical conductivity, by monitoring changes in the total magnetic field, sum of the excitation and reaction fields. We are interested in reproducing numerically this interaction. Knowing that in most applications, the eddy current sensor is moved over the tested material in order to check it (see Fig. 1), we present a particular domain decomposition approach to treat the magnetodynamic problem [1], involving finite element discretizations in overlapping subdomains which are coupled in the spirit of the mortar element methods [2,11]. This approach fulfills the requirements of

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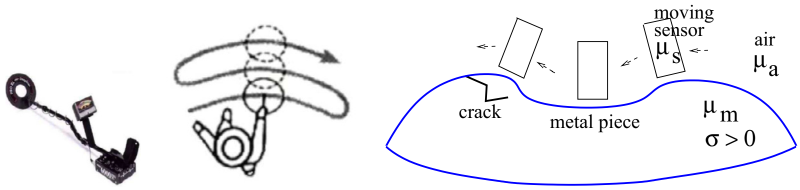

Fig. 1. From left to right: example of metal detector and of the screening technique to find metallic objects hidden in the soil (courtesy of Wikipedia). Problem configuration for eddy-current integrity testing of a metal piece not necessarily flat.

precision and flexibility that are needed to deal with such a kind of applications. Suitable weak coupling conditions replace the point-wise continuity at the interfaces of the decomposition.

The considered approach differs from other existing ones, such as, for example, the Arlequin method [6], the FEM with patches [9] or the fictions domain technique [8]. In the Arlequin approach, a super-imposition of different (mechanical) models is performed in a sub-zone of the whole domain by a weak and compatible gluing at the energy level. This results in an additional bilinear form over the sub-zone to be included in the initial variational problem and involves suitable weighted functions to make the transition from one mechanical model to the other. In the overlapping MEM, the physical model is the same, but the discretization of one (fixed) subdomain is extended under the discretization of the other (moving) subdomain with the minimal requirement of (weak) continuity of the physical solution at some interfaces. Moreover, the discrete solution with the overlapping MEM is computed directly without going through successive corrections in patches of finite elements as it occurs in [9] or other Schwarz type domain decomposition methods. In the fictitious domain technique [8], the initial problem to be solved in a complicated domain D is substituted by a new problem set in a simple shaped domain Ω which contains D with the additional condition that the solution computed in Ω tends to that of the initial problem when Ω shrinks to D. Similarly, in the overlapping MEM, the moving object (here the sensor-coil) with complicated shape can be included in a simple shape domain ω together with some air (the choice of ω is done by the user). The acceleration of the problem solution is more related to the fact that we do not have to re-mesh the domain Ω for any new position of ω rather than on the use of a special solver on Ω as in [8].

The paper is organized as follows. In Section 2 we describe the magnetodynamic problem in terms of potentials and its variational formulation. In Section 3, we introduce the discrete formulation of the continuous problem together with the coupling method. We discuss also on the numerical details of the proposed method and on the way the coupling condition has to be imposed in the final algebraic system. In Section 4, we recall the basic features of the EC NDT. This helps introducing notations that will be useful later in Section 5 to describe the obtained numerical results. Conclusions are then drawn in Section 6. The methodology is here presented in two dimensions for simplicity but extends straightforwardly to three dimensions. A preliminary formulation of the method, where the coupling constrains are imposed by means of Lagrange multipliers, has been discussed, for the time-harmonic Maxwell's equations: in [4] with constant coefficients and in [5] with jumps in the coefficients. In this paper, the formulation of the method, with the coupling constrains strongly imposed in the final system, is presented for the low-frequency time-dependent Maxwell's equations with jumps in the coefficients. The strong imposition of the constrains takes inspiration from [13] but is here generalized to situations where the reaction of the conductor has to be taken into account.

2. Problem setting and formulation

In EC NDT applications in $\mathbb{R}^3$, the applied current density $\mathcal{J}_s$ circulates in the sensor which moves but remains always in the insulating region. The electrically conducting tested material occupies a non-insulating region V_c close to the sensor. By analyzing the electromagnetic answer from the tested object, we may infer on its structural integrity. We thus look for $\mathcal{E}, \mathcal{H}, \mathcal{J}$ and $\mathcal{B}$ such that in V_c

$$\nabla \times \mathcal{H} = \mathcal{J}, \quad \nabla \times \mathcal{E} = -\partial_t \mathcal{B}, \quad \nabla \cdot \mathcal{B} = 0, \tag{1}$$

and in $\mathbb{R}^3 \setminus V_c$

$$\nabla \times \mathcal{H} = \mathcal{J}_s, \quad \nabla \cdot \mathcal{B} = 0. \tag{2}$$

In the equations above, $\mathcal{H}$ (resp. $\mathcal{B}$) is the magnetic field (resp. magnetic induction), $\mathcal{E}$ is the electric field and $\mathcal{J}$ is the conducting current density. Inductions and fields are linked by the constitutive properties, i.e., $\mathcal{B} = \mu\mathcal{H}, \ \mathcal{J} = \sigma\mathcal{E}$, where μ is the magnetic permeability and σ stands for the electric conductivity. We assume that the material parameters are associated with linear isotropic media (σ and μ are piece-wise constant functions) and that the source current $\mathcal{J}_s$ is divergence-free in $\mathbb{R}^3 \setminus V_c$.

Problem (1) and (2) is well-posed in $\mathbb{R}^3$ by adding regularity conditions at infinity (such as, from Biot–Savart's law, that $\mathcal{B}$ behaves as $1/r^3$ far from conductors) and suitable jump conditions for the fields' normal or tangential component, such as $[\mathcal{B}] \cdot n = 0$ or $[\mathcal{H}] \times n = \mathbf{0}$, at any interface where σ or μ are discontinuous (here, n is the unit normal to the interface and $[v]$