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An approximate Riemann solver for shallow water equations and heat advection in horizontal centrifugal casting

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ABSTRACT

An approximate Riemann solver was developed for solving modified shallow water equations (SWE) and energy transport describing the average flow dynamics of a single layer spreading inside a horizontally rotating cylinder. The numerical model was particularly developed for simulating the horizontal centrifugal casting (HSC) of the outer shell of a work roll. The SWE were derived in the rotating frame of reference; therefore, fictitious forces (the centrifugal force and the Coriolis force) were considered. In addition, other forces such as the bed shear force, the force of gravity, the wind shear force and forces arising from the variable liquid/solid interface were taken into account. The Jacobian matrix of the nonlinear hyperbolic system of PDEs was decomposed into a set of eigenvalues and corresponding eigenvectors using standard and corrected Roe averages. A Harten–Hyman entropy fix was used to prevent expansion shocks (entropy violating solutions) typically occurring during transonic rarefactions. Source terms were applied as a stationary discontinuity and were physically bounded and well-balanced for steady states (producing non-oscillatory solutions). Each wave was upwinded using the explicit Godunov's method. The high resolution corrections with flux limiters were used to achieve second order of accuracy and dispersion free solutions at discontinuities. In addition to the Riemann solver, a central scheme FV model was used to solve the heat diffusion inside the cylinder (mold) and partially solidified liquid layer, coupled with the solidification model. Several simulations were performed, results were analyzed and discussed.

1. Introduction

The present paper describes in detail an approximate Riemann solver for solving modified shallow water equations and energy transport of a single liquid layer spreading inside a horizontally rotating cylinder. The horizontal centrifugal casting (HSC) of the outer shell of a work roll is an industrial application of such a process. In brief, the HSC process (Fig. 1) can be summarized as the following: A cylindrical mold is horizontally placed on four carrying rollers, from which two coaxial are always driven, whereas other two are driving. While the mold is rotating at a high speed (~600 rpm), a liquid metal is poured from the crucible via the statically mounted runner approximately in the center of the mold. Due to high centrifugal forces the liquid metal spreads uniformly and creates a sleeve of a constant thickness. This particular process of casting a work roll

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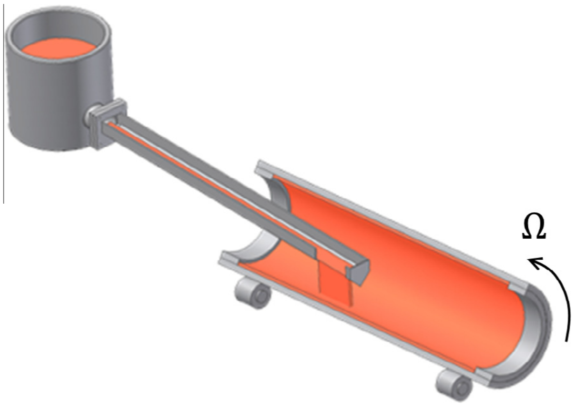

Fig. 1. A schematic of the horizontal centrifugal casting.

takes approximately 35 min. Generally, a centrifugally cast product has a superior mechanical properties compared to conventional gravitational castings [1].

Among other research papers published recently, mathematical models differ mainly in whether the flow was solved or not. Numerical studies solving the flow dynamics were mostly using the VOF method [2] to capture the interface between the liquid layer and the surrounding air. The simulations performed by Xu [3] were terminated at 30 s and thus; only the filling stage of the casting could be analyzed. In the paper, the time dependent distribution of the surface temperature on the external mold wall was obviously the main objective. The solidification model was not however mentioned in the text. Another interesting paper by Kaschnitz [4] presented a HSC simulation of seamless pipes performed using the commercial package FLOW3D. In order to avoid extremely low time steps, momentum equations were solved in the rotating frame of reference. However, due to a very small wall-to-length thickness ratio, one simulation still took considerably long time (~20 days). A commercial code (STAR-CD V4) was used also in a work done by Prasad [5]. The mesh inside the mold was entirely constructed out of rather coarse polyhedral elements, which allowed notably large time steps (~0.01 s). Only the continuity and momentum equations were solved for the flow. Heat transfer and solidification were not discussed in the paper. Results from simulations showed roughly how the melt is spreading during the filling stage, however no details are given on how the filling was imposed and whether the model could capture some free surface patterns or not. It can be concluded that such multiphase (VOF) simulations can successfully resolve a flow field of the liquid metal during the HSC process, however; only a limited period of time is usually concerned. Moreover, these simulations are solely covering the topic of solidification of the liquid layer. On the other hand, several research papers can be found dealing with the complete solidification of the liquid layer yet omitting the flow. The main object of consideration is a time dependent thickness of the solidifying shell often influenced by a segregation of some element due to a density difference and extremely high centrifugal pressure. For example in [6], Drenchev introduced a numerical model discussing some aspects of macrosegregation of reinforcing particles in a metal matrix. The enthalpy equation was the primary equation to solve with thermal physical properties determined from the segregation model. Since the flow (or the mold filling) was not included, the initial thickness of the liquid layer was uniform and identical to the final thickness of the shell. Similar numerical models can be found in [7,8]. The main bottleneck is the fact that the model lacks variances in the mold and shell temperatures due to the localized filling, which in turn affects the local thickness of the solidified shell and the macrostructure pattern consequently.

In the present paper, a fruitful effort was made to develop a novel approach, which would take the flow into consideration and still allow for a complete solidification of the shell in a reasonable computational time. The flow during the HSC process can be characterized as a free surface flow, in which the thickness of the liquid layer is rather small compared to the length of the mold. For this reason, it is rational to expect the momentum in the radial direction to be negligible compared to the momentum in the axial and tangential direction. Taking the 3D Euler equations and the continuity equation leaving out the momentum in the radial direction, integrating momentum and mass equations along the liquid height, and applying kinematic boundary condition on the free surface one obtains the 2D shallow water equations (SWE) originally derived in [9]. From the asymptotic series of the static pressure only the first term, the hydrostatic pressure is considered and terms with higher derivatives are neglected. This as a hydrostatic condition is a leading order approximation to the static pressure and is relevant for flows where a horizontal scale L is large compared to a characteristic height H. Note that no assumption is made about amplitudes of waves on the free surface. All the nonlinearities are retained. The original SWE are strictly hyperbolic nonlinear PDEs. In the following text, the SWE are modified to describe the average flow dynamics of the liquid layer inside the horizontally rotating cylinder. Next, an approximate Riemann solver is derived and carefully detailed. Several 1D numerical tests are shown in order to demonstrate the capability of the Riemann solver. In addition, 2D numerical examples are presented showing the simulation of the HSC process. Note that the approximate Riemann solver is used to solve the SWE and the heat advection within the liquid layer. An additional central difference FV model is used to calculate the heat