Applied Mathematics and Computation 267 (2015) 427–435

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Coupled shrinkage and damage analysis of autoclaved aerated concrete

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ARTICLE INFO

Keywords:
Shrinkage
Moisture transport
Damage
Autoclaved aerated concrete

ABSTRACT

This paper is devoted to analysis of shrinkage and damage of autoclaved aerated concrete. Coupled hydro-thermo-mechanical analysis is used for detailed description of drying which causes the shrinkage and damage consequently. The heat and moisture transfer are fully coupled while the staggered approach is used between transports and mechanics. Material parameters were obtained from laboratory experiments and the results of numerical simulations correspond with measured data.

1. Introduction

The number of advanced materials used in civil engineering structures is gradually increasing. This paper is devoted to modelling of autoclaved aerated concrete (AAC). Motivation of the usage of the AAC is based on its suitable material properties such as small density and small thermal conductivity. Advantages of the AAC is also low price and environmental friendliness. On the other hand, the AAC suffers by lower strength or large shrinkage caused by drying which is usually accompanied by creation of cracks or voids.

The cracks are created between the AAC blocks and plaster or even in the blocks themselves. Such damage influences transport processes in the concrete and the transport of various chemical species could start or is accelerated. The cracks are caused by significant volume changes because of drying or wetting and temperature variation. Drying shrinkage presents the most important volumetric change during the lifetime of any AAC construction. The rate of drying shrinkage depends on the composition of AAC. It has been reported in [1] that drying shrinkage of AAC with only cement as a binder is significantly higher than that produced with lime or lime and cement. It has been concluded in recent research [2] that drying shrinkage of different types of AAC is increasing with decreasing moisture content in the AAC pore system and this increase is fastest in the range of very low moisture, much lower than 6%.

2. Mechanical model

Mechanical model is formulated as a time dependent model because it will be coupled with moisture transfer. Small strains are assumed and therefore the additive strain decomposition in the rate form reads

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http://dx.doi.org/10.1016/j.amc.2015.02.016
0096-3003/

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{sh}_{ij}, \quad (1)$$

where $\dot{\varepsilon}_{ij}$ is the rate of change of the total strain tensor, $\dot{\varepsilon}^{e}_{ij}$ is the rate of change of the elastic strain tensor, $\dot{\varepsilon}^{sh}_{ij}$ is the rate of change of the strain caused by shrinkage. The shrinkage strain depends mainly on the amount of moisture removed from material which is described by the volumetric moisture content w (m^3/m^3). Let a one-dimensional bar with the length l be assumed together with a change of the volumetric moisture content Δw. The change of the bar length, Δl, has the form

$$\Delta l = l(w + \Delta w) - l(w) \approx \frac{\partial l}{\partial w}\Delta w. \quad (2)$$

In the case of $\Delta w > 0$, swelling occurs and the length grows while for $\Delta w < 0$ shrinkage takes place and the length of bar reduces. The change of shrinkage/swelling strain is defined

$$\Delta \varepsilon^{sh} = \frac{\Delta l}{l} = \frac{1}{l}\frac{\partial l}{\partial w}\Delta w = \beta \Delta w, \quad (3)$$

where β is the hygric expansion/shrinkage coefficient (–). Unlike the thermal strain depends linearly on the change of temperature because the thermal expansion coefficient is constant for many materials, the shrinkage/swelling strain depends nonlinearly on the change of the volumetric moisture content and the hygric expansion/shrinkage coefficient cannot be assumed constant. Experiments [2] with AAC blocks showed an isotropic character of the shrinkage and therefore the change of shrinkage/swelling strain tensor have the form

$$\Delta \varepsilon^{sh}_{ij} = \beta \Delta w \delta_{ij}, \quad (4)$$

where δ_{ij} is the Kronecker delta. The time derivative of the shrinkage strain components has the form

$$\dot{\varepsilon}^{sh}_{ij} = \beta \dot{w} \delta_{ij}. \quad (5)$$

The hygric expansion/shrinkage coefficient should be obtained from measured data on a one-dimensional specimen in the form

$$\beta = \frac{\Delta l}{l \Delta w} = \frac{\Delta \varepsilon}{\Delta w}. \quad (6)$$

AAC belongs to quasi-brittle materials which are characterized by evolution of defects such as microcracks and microvoids when a certain strain level is exceeded. If the evolution of strains continues, the growth of defects localizes to some of them while evolution of the rest stops. The process is called the localization of inelastic strains and many models can be used for its description with various concepts of yielding. The scalar isotropic damage model is one of the simplest and most popular models of continuum damage mechanics. More details about this model can be found in [3] or [4]. The dimensionless damage parameter ω is defined with help of cross section area of defects A_d and the total cross section area A as follows

$$\omega = \frac{A_d}{A}. \quad (7)$$

The evolution law which connects the damage parameter ω and the strain could be expressed in the form

$$\omega = g(\tilde{\varepsilon}), \quad (8)$$

where g is a function which has to be established experimentally and $\tilde{\varepsilon}$ is the equivalent strain. In one-dimensional case, the equivalent strain is equal to the one-dimensional strain but generally it is given by a formula mapping the strain tensor components to a single value. As an example can serve the Euclidean norm

$$\tilde{\varepsilon} = \sqrt{\varepsilon_{ij}\varepsilon_{ij}}, \quad (9)$$

where ε_{ij} is the total strain tensor. The Einstein summation convention is used in tensorial notation. For concrete, the equivalent strain could be defined by the Mazars norm [5] which has the form

$$\tilde{\varepsilon} = \sqrt{\langle \varepsilon_\alpha \rangle \langle \varepsilon_\alpha \rangle}, \quad (10)$$

where ε_α denotes the principal values of the strain tensor and the symbol $\langle\rangle$ denotes selection of positive components (the Macaulay brackets). With respect to thermodynamic laws, the damage parameter cannot decrease. Additional variable κ is defined and it expresses the maximum equivalent strain ever reached. The evolution law (8) has to be rewritten to the form

$$\omega = \begin{cases} g(\tilde{\varepsilon}), & \kappa = \tilde{\varepsilon} \quad \text{if } \kappa < \tilde{\varepsilon}, \\ g(\kappa), & \qquad\quad \text{if } \kappa \geqslant \tilde{\varepsilon}. \end{cases} \quad (11)$$

Damage models lead to responses which are mesh sensitive [3]. It is connected with the dissipated energy which depends on the characteristic size of damaged elements in numerical simulation. If the characteristic size of elements tends to zero, the dissipated energy tends to zero too which is not physically realistic result. The dissipation occurs in a zone of which the