Applied Mathematics and Computation 267 (2015) 498–505

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical simulation of circumferentially averaged flow in a turbine

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ARTICLE INFO

Keywords:
Turbine
Finite volume method
Loss model

ABSTRACT

The paper refers about the development of a fast computational code, which should be able to provide an approximate information about the three-dimensional flow field in a multi-stage turbine. The code is based upon the solution of circumferentially averaged Euler equations coupled with the thermodynamic, geometry and loss prediction models. The computational domain is the meridional cut of a turbine. The Euler equations are solved by a finite volume solver with the AUSM type flux. Initial tests showed, that developed solver is able to predict well radial distributions of flow parameters upstream and downstream considered blade cascades at a fraction of CPU time compared to fully three-dimensional simulations.

1. Introduction

The paper refers about the development of a fast computational code intended for early stages of turbine design. There exist different methods ranging from quasi 1D solvers [1] to solvers based on the circumferentially averaged Euler [2] or Navier–Stokes [3] equations. The aim of this work is to develop a code, which is able to provide an approximate information about the three-dimensional flow field in a multistage turbine. It is based upon the solution of steady circumferentially averaged flow. The computational domain is two-dimensional. It is defined by the meridional cut of a turbine, i.e. the real shapes of hub and tip casing are parts of the domain boundary. The shapes of the pressure and the suction surfaces of the blade, which cannot be described by the definition of the computational domain, are included in the form of additional parameters. The variable thickness of blades is taken into account using the channel width parameter. The shape of the blades is approximated by the shape of the midplane, which is located in the middle between the suction and the pressure surfaces of the blade. This plane is implemented in the form of normal vectors. Similarly to [2] the flow is modeled by the circumferentially averaged Euler equations, which are coupled with geometry and loss models. The solution domain is discretized by a structured grid. The flow solver uses a finite volume method with AUSM (Advection Upstream Splitting Method) type flux [7]. Initial tests show, that developed solver is able to predict well the radial distributions of flow parameters upstream and downstream the considered blade cascades at short CPU time compared to fully three-dimensional simulations. The used finite volume method has advantage over previous methods based on a streamlines and stream functions, which did not work well for transonic speeds and they did not generally guarantee the conservation of mass, momentum and energy.

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http://dx.doi.org/10.1016/j.amc.2015.01.086
0096-3003/

2. Flow model

Consider the system of cylindrical coordinates r, φ and z, which is fixed to a single blade row of axial turbine. The z axis coincides with the axis of the cascade. The cascade can rotate around its axis with angular velocity ω. The Euler equations in the considered system of coordinates read

$$\frac{\partial \widetilde{W}}{\partial t} + \frac{1}{r}\frac{\partial (r\widetilde{F})}{\partial r} + \frac{1}{r}\frac{\partial \widetilde{G}}{\partial \varphi} + \frac{\partial \widetilde{H}}{\partial z} = \widetilde{B}, \tag{1}$$

$$\widetilde{W} = \begin{bmatrix} \rho \\ \rho v_r \\ \rho v_\varphi \\ \rho v_z \\ e \end{bmatrix}, \quad \widetilde{F} = \begin{bmatrix} \rho v_r \\ \rho v_r^2 + p \\ \rho v_r v_\varphi \\ \rho v_r v_z \\ v_r(e+p) \end{bmatrix}, \quad \widetilde{G} = \begin{bmatrix} \rho v_\varphi \\ \rho v_\varphi v_r \\ \rho v_\varphi^2 + p \\ \rho v_\varphi v_z \\ v_\varphi(e+p) \end{bmatrix}, \quad \widetilde{H} = \begin{bmatrix} \rho v_z \\ \rho v_z v_r \\ \rho v_z v_\varphi \\ \rho v_z^2 + p \\ v_z(e+p) \end{bmatrix},$$

$$\widetilde{B} = \begin{bmatrix} 0 \\ \rho \frac{v_\varphi^2}{r} + \frac{p}{r} + \rho r \omega^2 + 2\rho\omega v_\varphi \\ -\rho \frac{v_r v_\varphi}{r} - 2\rho\omega v_r \\ 0 \\ 0 \end{bmatrix},$$

where the symbol t denotes time, ρ density, v_r, v_φ and v_z velocity components, e the total energy per unit volume and p the pressure. We consider the steady solution, therefore we might take a single blade passage (i.e. blade-to-blade channel together with sufficiently long upstream and downstream parts) as the solution domain D. The ranges for the coordinates z and r are defined by the domain D_{zr} (projection of D onto zr plane). The range for the coordinate φ is limited by $\varphi_1(r,z)$ and $\varphi_2(r,z)$. The idea of circumferential averaging is equivalent to the finite volume discretization of D with a single layer of cells in the tangential direction. Consider an arbitrary cell of this discretization defined by $V \subset D_{zr}$ and $\varphi_1(r,z) < \varphi < \varphi_2(r,z)$. The integral form of the Eq. (1) then reads

$$\iint_V \int_{\varphi_1}^{\varphi_2} \left(\frac{\partial \widetilde{W}}{\partial t} + \frac{1}{r}\frac{\partial (r\widetilde{F})}{\partial r} + \frac{1}{r}\frac{\partial \widetilde{G}}{\partial \varphi} + \frac{\partial \widetilde{H}}{\partial z} \right) r d\varphi dr dz = \iint_V \int_{\varphi_1}^{\varphi_2} \widetilde{B} r d\varphi dr dz. \tag{2}$$

Swapping the derivative and the integration with respect to φ and using the mean value theorem for the left hand side of the Eq. (2) yields

$$\iint_V \left(\frac{\partial (brW)}{\partial t} + \frac{\partial (brF)}{\partial r} + \frac{\partial (brH)}{\partial z} + (F_1, G_1, H_1)\vec{n}_1 - (F_2, G_2, H_2)\vec{n}_2 \right) dr dz,$$

where $b(r,z) = \varphi_2(r,z) - \varphi_1(r,z)$ is the width of blade channel and V is the finite volume of the discretization of D_{zr}, see examples in the Figs. 2 or 10(b). The vectors W, F, G and H are averages of $\widetilde{W}, \widetilde{F}, \widetilde{G}$ and $\widetilde{H}$ with respect to φ. The vector F_i is the vector $\widetilde{F}$ for $\varphi = \varphi_i$ and

$$\vec{n}_i = \left(\frac{\partial \varphi_i}{\partial r}, -\frac{1}{r}, \frac{\partial \varphi_i}{\partial z} \right), \tag{3}$$

is the normal vector to the surface $\varphi = \varphi_i(r,z)$. The surfaces $\varphi = \varphi_i(r,z)$ coincide partly with the blade surface. Therefore we need to apply the non-permeability condition in the form $(F_i, G_i, H_i)\vec{n}_i = [0, p_i\vec{n}_i, 0]^T$ on the part of $\varphi = \varphi_i(r,z)$ surfaces and for the remaining part to apply the periodicity conditions, i.e. $(F_1, G_1, H_1) = (F_2, G_2, H_2)$, $\vec{n}_1 = \vec{n}_2$ and $p_1 = p_2$. The non-permeability condition yields

$$(F_2, G_2, H_2)\vec{n}_2 - (F_1, G_1, H_1)\vec{n}_1 = p_2 \begin{bmatrix} 0 \\ r\partial\varphi_2/\partial r \\ -1 \\ r\partial\varphi_2/\partial z \\ 0 \end{bmatrix} - p_1 \begin{bmatrix} 0 \\ r\partial\varphi_1/\partial r \\ -1 \\ r\partial\varphi_1/\partial z \\ 0 \end{bmatrix}. \tag{4}$$

The above substitution (4) can be used also in the case of periodicity conditions, where both sides turn to zero. The circumferentially averaged integral form of governing equations is therefore

$$\iint_V \frac{\partial (brW)}{\partial t} dz dr + \iint_V div(brH, brF) dz dr = \iint_V brQ dz dr, \tag{5}$$