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Modeling nonlinear phenomena in deforming fluid-saturated porous media using homogenization and sensitivity analysis concepts

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ABSTRACT

Homogenization of heterogeneous media with nonlinear effects leads to computationally complex problems requiring updating local microstructures and solving local auxiliary problems to compute characteristic responses. In this paper we suggest how to circumvent such a computationally expensive updating procedure while using an efficient approximation scheme for the local homogenized coefficients, so that the complexity of the whole two-scale modeling is reduced substantially. We consider the deforming porous fluid saturated media described by the Biot model. The proposed modeling approaches are based on the homogenization of the quasistatic fluid–structure interaction whereby differentiation with respect to the microstructure deformation is used as a tool for linearization. Assuming the linear kinematics framework, the physical nonlinearity in the Biot continuum is introduced in terms of the deformation-dependent material coefficients which are approximated as linear functions of the macroscopic response. These functions are obtained by the sensitivity analysis of the homogenized coefficients computed for a given geometry of the porous structure which transforms due to the local deformation. The deformation-dependent material coefficients approximated in this way do not require any solving of local microscopic problems for updated configurations. It appears that difference between the linear and nonlinear models depends significantly on the specific microstructure of the porous medium; this observation is supported by numerical examples.

1. Introduction

The Biot model of poroelasticity [1] proposed originally using the phenomenological approach has been become the subject on many works re-examining this model in the framework of the homogenization theory. A formal two-scale asymptotic technique has been employed in [2,3] and extended further to double porosity media in [4,5], see also [6]. The large body of papers were devoted to the acoustic properties of the fluid-saturated porous media, we shall focus on quasistatic flows in saturated elastic porous media with periodic structures. Recently interaction of such media with elastic bodies was considered in [7], where a special scaling of the acceleration terms resulting from the dimensional analysis under the

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slow flow assumptions was considered. The model obtained in that paper is consistent with the one derived in [8] where the viscoelastic solid was considered under the assumption of slow quasistatic processes. This justified an uncoupled upscaling of the fluid–structure interaction: the homogenization of the static loading problem led to the effective poroelastic coefficients, whereas the permeability tensor was given separately by conventional upscaling of the Stokes flow in the rigid porous structure [15]. We shall adhere this approach in the present paper dealing with quasistatic flows in saturated elastic porous media. The linear poroelasticity model is classical, see e.g. [7], here derived using the homogenization of the elastic solid skeleton with periodic pores saturated by a slightly compressible static fluid [6]. Our main new contribution is in the extension of the linear model by capturing influence of the deforming porous structure on the effective properties.

Although, in the context of the homogenization method, the linear models of porous fluid-saturated media has been studied in many papers, the issues of nonlinear effects and large deforming media in particular have not received much attention so far [9–11]. Besides the complexity of the homogenization of the nonlinear problem, cf. [12], the implementation of such model leads to prohibitively expensive computations because of the need for solving local problems at all integration points of the macroscopic domain. Quite recently in [11] the arbitrary Lagrangian–Eulerian formulation was used and the homogenized model was obtained by the formal two-scale method while linearizing the Piola tensor; some simplifications of the model were suggested to prove the concept of solving the nonlinear problem efficiently. The dependence of the permeability on the pressure, stress and deformation fields has been investigated in a number of papers, see e.g. [13,14].

In the present paper we derive a physically nonlinear model of the Biot continuum. For this we combine the homogenization method and the sensitivity analysis of the homogenized material coefficients with respect to microstructure deformation. On one hand, this approach introduces an approximation of the fully nonlinear model of poroelasticity, on the other hand, it leads to an efficient computational algorithm. As the main advantage, the effective coefficients depend directly on the macroscopic strain and pressure, so that solving the local problems in updated deformed configurations is avoided unlike the fully nonlinear two-scale problem, see e.g. [10] where a case of large deforming medium with fluid inclusions was treated. Here we consider quasistatic flows in saturated porous media with periodic structures, although this assumption can be alleviated to deal with only locally periodic structures. The Biot-type poroelasticity model is proposed to treat situations when the deformation has a significant influence on the permeability tensor controlling the seepage flow and on the other poroelastic coefficients. To respect dependence of the effective properties on the microstructure deformation, we proposed to use the Taylor expansion with respect to the macroscopic variables involved in the global problem. For this, the sensitivity analysis well known from the shape optimization is adopted [16]. The resulting nonlinear formulation involves the effective poroelasticity and permeability coefficients which are linear functions of the macroscopic response, cf. [17].

Under the small deformation assumption (the linear kinematics) and the first order gradient theory of the continuum, the constitutive laws are considered usually in linearized forms involving material constants independent on the field variables, like deformation, or stress. In this context, our treatment of the material coefficients depending on the stress and deformation state can be viewed as an extension of the first order theory, whereby the linear strain kinematics still holds and the initial domain is taken as the reference.

The paper is organized, as follows. In Section 2 we present the linear Biot–Darcy continuum model obtained by homogenization; the local problems for the characteristic response functions and the formulae for the homogenized coefficients are given. To introduce linear expansions of the homogenized coefficients, in Section 3 we describe their sensitivity analysis with respect to the microstructure perturbation. The so-called weakly nonlinear model is introduced in Section 4 where also the linearization scheme is described which enables to compute iteratively an equilibrium state for any time level with a given time step; some auxiliary technical calculations are postponed in the Appendix A. In numerical examples reported in Section 5 we focus on comparing the linear and the weakly nonlinear models.

Through the paper we shall adhere to the following notation. The position x in the medium is specified through the coordinates (x_1, x_2, x_3) with respect to a Cartesian reference frame. As usually, the vectors will be denoted by bold letters, for instance, $\boldsymbol{u}(x)$ denotes the solid matrix displacement vector field depending on the spatial variable x; components of this vector will be denoted by u_i for $i = 1, \ldots, 3$, thus $\boldsymbol{u} = (u_i)$. The partial derivatives with respect to x_i are denoted by ∂_i, $\nabla = (\partial_i)$ is the gradient operator. We shall also use the microscopic (dilated) Cartesian reference system of coordinates (y_1, y_2, y_3), therefore the abbreviations $\partial_i^x = \partial/\partial x_i$ and $\partial_i^y = \partial/\partial y_i$ will be employed alternatively. The Einstein summation convention is employed. The inner product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is $\boldsymbol{a} \cdot \boldsymbol{b} = a_i b_i$. In analogy, $\boldsymbol{e} : \boldsymbol{e}' = e_{ij} e'_{ij}$. In particular $\nabla \cdot \boldsymbol{a} = \partial_i a_i$ is the divergence of $\boldsymbol{a}$. The time derivative is denoted by dot, thus, $\dot{a} = \frac{\mathrm{d}a}{\mathrm{d}t}$. By $\partial_b a \circ \delta b$ we mean the differential of a (depending on b) by the perturbation δb.

2. Linear homogenized Biot continuum

Assuming slow flows through a slightly deforming porous structure undergoing quasistatic loading of the solid phase by external forces, the upscaled medium is described by the Biot–Darcy coupled system of equations which was derived in [3] using a two-scale technique. Consistently with results of [7,11,8], the effective material parameters of the model can be derived by the homogenization of two decoupled problems: (1) deformation of a porous solid saturated by a slightly compressible static fluid and (2) Stokes flow through the rigid porous structure. Therefore, in the next subsections we present the homogenized model which was obtained by the asymptotic analysis with respect to the positive dimensionless scale